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# Imperfect Information Flow of Agents Communication in Arrow Logic

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**Abstract:** This paper formalizes the communication of agents with modal operators in arrow logic. A communication between agents consists of an agent's utterance and the other agent's perception, thus, both of the utterance and the perception are regarded as parts of a communication channel between agents. Information is regarded as a propositional content of a sentence. An information channel where information flows can be considered to be a program, in the sense that it gets an utterance as an input and puts an output to be a perception of some agent. In the real situations, there are so called miscommunications. Thus, the communication channel as a program may add some noise on information indeterministically. We implement the noises are some modal operators on information. We try to formalize the communication channels in arrow logic. In that, we especially pay attention to the following three problems: *channel bottleneck*, *unreliable channel*, and *reverse information*. This paper's contribution is two-fold. First, we formalize the theory of information flow, based on situation semantics, in terms of arrow logic. Secondly, we propose the theory of communication channels between agents by using arrow logic, where, classical modal operators like *knowledge*, *belief*, and *perception* are distributed on various places on the communication channel. We discuss the satisfiability and the applicability of our formalization, using the test principles by Barwise on this information flow model.

**Key Words:** arrow logic, agent, communication, modal operator, information flow, belief

**Category:** I.2.0

## 1 Introduction

Over the last few years, a lot of studies have been made on the logical formalization and the theory of rational agents ([Moore 1985; Wooldridge and Jennings 1995; Wooldridge, Muller, and Tambe 1996; Muller, Wooldridge, and Jennings 1997; van Linder, van der Hoek, and Meyer 1997], and so on). According to [Shoham and Cousins 1994], there are two main approaches in this field: one is an informational aspect such as epistemic modal logic like logic of knowledge and belief [Halpern and Moses 1992], and the other is a motivational aspect such as commitments and obligations ([Cohen and Levesque 1986; Cohen and Levesque 1990a], and so on). These formalizations mainly concern how agents act and/or change their internal states after they receive information. They stand on the possible world semantics that is one of the demonstrative semantics. On the other hand, there is quite a bit of study on the formalization of agents communication([Cohen and Perrault 1979; Cohen and Levesque 1990b], and so on) which are based on the speech act theory by Austin[Austin 1962] and Searle[Searle 1969]. On these studies, illocutionary acts are regarded as the operations on belief and desire. In order to formalize the multi-agents system, we should consider not only knowledge and belief of agents but also the communication between agents. Thus, we propose a formalization of communication between agents as information flow [Barwise, Gabbay, and Hartonas 1996] formalized by arrow logic [van Benthem 1994; Marx, Polos, and Masuch 1996]. To realize neat and clear formalization of imperfect information flow, we adopt arrow logic and propositional dynamic logic with arrows [van Benthem 1996].

In the following section, first we show the nature of belief and perception in natural language sentences and the robot navigation example for our model. Next we formalize information flow using arrow logic and show five test principle of Barwise [Barwise 1993] with arrows. Then we propose a model of the information flow between agents. Thus, agents who have knowledge and belief, communicate with other agents, are modeled from the information theoretic point of view [Barwise, Gabbay, and Hartonas 1996]. It is based on the situation semantics[Barwise 1989; Devlin 1991] focuses on information transmitted by language. Thereafter, we apply the test principles of Barwise [Barwise 1993] to our communication model, and show the following three problems: (1)Channel bottleneck, (2)unreliable channel, and (3)reverse information. Finally, we discuss the applicability of our model and conclude our research.

In [van Linder, van der Hoek, and Meyer 1997], the organization of believes in agents' internal states is examined. The channel algebra which is almost the same as the arrow logic is proposed in [Moss and Seligman 1994]. Our approach is similar to [van Linder, van der Hoek, and Meyer 1997] and [Moss and Seligman 1994], however, we mainly pay attention to imperfect information flow.

## 2 Problems of Agent Communication

The agent is the technical term used in very different meanings in accordance with application areas. One extreme is human beings, and in that case, information becomes natural language sentences. On the contrary, when agents are robots, the information becomes commitments or commands between agents. In this section, we show the problems of agent communication, from these two different areas.

### 2.1 Human Language Communication

At the beginning, we will examine the nature of two modal operators: belief and perception that appear in natural languages, based on [Devlin 1991]. A logical nature of mental attitudes is different in systems. In the typical epistemic logic, the belief operator is defined K45 or KD45 system:

$$B\Phi \not\supset \Phi,$$
$$B(\Phi \wedge \Psi) \overset{\supset}{\subset} (B\Phi \wedge B\Psi),$$

$$B(\Phi \vee \Psi) \overset{\supset}{\underset{\not\subset}{}} (B\ \Phi \vee B\ \Psi),$$

where '$\supset$' is the logical implication and $\Phi$ and $\Psi$ are propositional contents of natural language.

We basically obey these natures of belief modality, however, we add a slight change according to the usage of natural languages, for the purpose of applying it to the agent communication. We define the following operators:

$B_a\Phi$: agent $a$ believes $\Phi$,
$P_a\Phi$: agent $a$ perceives $\Phi$.

We cannot claim the veridicality of a sentence if it is accompanied by $B$ operator. Also, we cannot decompose such a $B$ sentence into logical 'or' sub-sentences. Below are the examples by Devlin [Devlin 1991]. (We omit the subscript of agent name in operators in the following examples, for readability.)

> (1) "That Naomi believes Melissa ate the cookie does not entail that Melissa actually did eat the cookie."
>
> (2) "Naomi can believe we will spend the night in Monterey or we will spend the night in Carmel, without believing that we will spend the night in Monterey and without believing we will spend the night in Carmel."
>
> (3) The agent may not connect $\Phi$ and $\Psi$ in any way in her mind, so $\Phi \wedge \Psi$ $(\Phi \vee \Psi)$ may be something toward which she has no attitude at all.

These examples are formalized in the following way:

(1)$'$ $B\Phi \not\supset \Phi$,
(2)$'$ $B(\Phi \vee \Psi) \not\supset (B\ \Phi \vee B\ \Psi)$,
(3)$'$ $B(\Phi \wedge \Psi) \not\subset (B\ \Phi \wedge B\ \Psi)$,
(3)$''$ $B(\Phi \vee \Psi) \not\subset (B\ \Phi \vee B\ \Psi)$.

In the similar way, we have examined natural language sentences including $B$ and $P$ operators, and their logical features are summarized as in the following table.

| | Veridicality | Conjunction | Disjunction |
|---|---|---|---|
| Belief | $B\Phi \not\supset \Phi$ | $B(\Phi \wedge \Psi) \supset (B\Phi \wedge B\Psi)$ | $B(\Phi \vee \Psi) \not\supset (B\ \Phi \vee B\ \Psi)$ |
| | | $B(\Phi \wedge \Psi) \not\subset (B\Phi \wedge B\Psi)$ | $B(\Phi \vee \Psi) \not\subset (B\ \Phi \vee B\ \Psi)$ |
| Perceive | $P\Phi \supset \Phi$ | — | — |

### 2.2 Robot Language Communication

We consider the concrete application of our formalization: a robot navigation with symbol by [Shibata et al. 1997] [see Fig.1]. The sender may be a human or a robot, and the receiver is a navigated robot. The sender recognizes a situation in the real world and symbolizes it. Therefore, The sender gives or sends the receiver a command constructed by some formal language. The receiver receives the command and does actions in the real world for executing the command. However, since sometimes they make mistakes, there is a gap between the sender's recognition and the receiver's. We want to explain this by the formalization of communication between agents. In the following section, we will use this application to explain our theories.

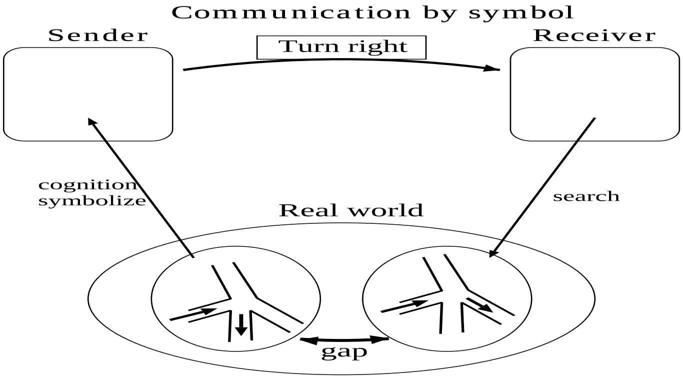

**Figure 1:** Robot Navigation

## 3 Information Flow in Arrow Logic

In this section we formalize the information flow [Barwise 1993] by dynamic arrow logic [van Benthem 1994; Marx, Polos, and Masuch 1996] and propositional dynamic logic with arrows [van Benthem 1996].

### 3.1 Arrow Logic - an Introduction

First, we introduce dynamic arrow logic and propositional dynamic logic in short, according to [van Benthem 1994].

**Arrow Logic** Arrow Frames are tuples $(A, C^3, R^2, I^1)$ with
$A$ a set of objects ('arrows') carrying three predicates:
$C^3 a, bc$ $a$ is a 'composition' of $b$ and $c$
$R^2 a, b$ $b$ is a 'reversal' of $a$
$I^1 a$ $a$ is a 'identity' arrow
Arrow Models $M$ add a propositional valuation $V$:
$M, a \models p$ iff $a \in V(p)$
$M, a \models \neg\phi$ iff *not* $M, a \models \phi$
$M, a \models \phi \wedge \psi$ iff $M, a \models \phi$ *and* $M, a \models \psi$
$M, a \models \phi \bullet \psi$ iff *there exist b,c with Ca,bc and* $M, b \models \phi,\ M, c \models \psi$
$M, a \models \phi^{\vee}$ iff *there exist b with Ra,b and* $M, b \models \phi$
$M, a \models Id$ iff $Ia$

**Dynamic Arrow Logic** Dynamic arrow logic adds one infinitary operator to the above language:
$M, a \models \phi^*$ iff *a can be C-decomposed into some finite sequence of arrows satisfying* $\phi$ *in* $M$

**Propositional Dynamic Logic with Arrows** A propositional dynamic logic, based on the arrow logic, is the two-level system: 'arrow talk' and 'state talk'.

Thus, a Boolean propositional language and two mechanisms of interaction are added between the following operations[van Benthem 1991]:
$M, x \models D\pi \quad$ iff $M, \langle x, x\rangle \models \pi$
$M, \langle x, y\rangle \models L\phi$ iff $M, x \models \phi$
$M, \langle x, y\rangle \models R\phi$ iff $M, y \models \phi$
where $a = \langle x, y\rangle$ says that, suppose there exist both end-points of the arrow $a$, $x$ is a left end-point of the arrow $a$ and $y$ is a right end-point of the arrow $a$. However, we cannot always indicate the both end-points of arrows.

### 3.2 The Theory of Information Flow

Besides the arrow logic, we need to introduce one more background theory to this paper, that is, the theory of information flow. First, we explain a *constraint* and an *information flow*. "John sees that Tom kisses Mary, and John tells it Lisa." This scene is constructed by the next five situations.
"Tom kisses Mary."
"John sees the scene that Tom kisses Mary."
"John knows that Tom kissed Mary."
"John tells Lisa that Tom kissed Mary."
"Lisa knows that Tom kissed Mary."

The information 'Tom kissed Mary' flows through these situations. This information flow is caused by some regular relations, called constraints, between these situations.

#### 3.2.1 Definition

The constraint regulates in types of situations, not in individual situations. A type of situation is abstracted from multiple individual situations. The flow of information has been formalized as a constraint in two situation types:

$$\begin{tikzcd} t_0 \arrow[r, Rightarrow] \arrow[d, "f"'] & t_1 \\ s_0 & \end{tikzcd} \qquad \qquad \begin{tikzcd} t_0 \arrow[r, Rightarrow] \arrow[d, "f"'] & t_1 \arrow[d, "f"'] \\ s_0 \arrow[r, squiggly] & s_1 \end{tikzcd}$$

When a situation type $t_0$ is anchored to a concrete situation $s_0$ by $f$, then this constraint is activated as an information flow, viz., $S_1$ is also anchored to a concrete situation $s_1$ by the same $f$ and information flows through a channel from site $s_0$ to site $s_1$ [Barwise 1993]. Our objective here is to regard this information flow as communication between agents. As information through a channel can be considered as propositional contents of natural language sentences, the information can be a proposition with modal operators formalized in the previous section.

According to the above diagram, the relation of carrying information has four arguments: the signal site, the target site, the indicating type, and the indicated type. We rewrite this as follows.

**Information Flow:** $s_1 : t_1 \longrightarrow s_2 : t_2$, where $s_1$'s being of type $t_1$ carries the information that $s_2$ is of type $t_2$. □

Using the robot navigation example, we explain information flow [see Fig.2]. The sender recognizes a situation in the real world and symbolizes it (anchor). And the sender gives or sends the receiver a command constructed by some formal language (speech). The receiver receives the command and does actions in the real world, executing the command (anchor, act). We can consider there is a constraint (speech-act) between the command and the action. However, since sometimes they make mistakes, there might be a gap between the sender's recognition and the receiver's.

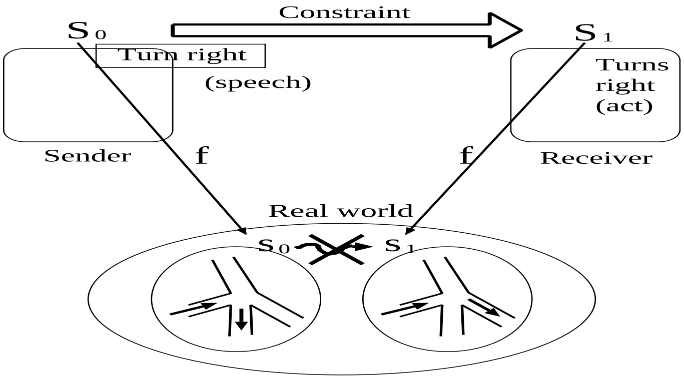

**Figure 2:** Application of our theory

### 3.2.2 Five Test Principles

Barwise proposed five test principles (Xerox principle/ Logic as information flow/ Addition of information/ Exhaustive cases/ Contraposition) that should be required as features of information flow [Barwise 1993]. His five test principles are defined as the following inference rules.

**(1)Xerox Principle**

$$\frac{s_1 : t_1 \longrightarrow s_2 : t_2 \quad s_2 : t_2 \longrightarrow s_3 : t_3}{s_1 : t_1 \longrightarrow s_3 : t_3}$$

If $s_1 : t_1$ carries the information that $s_2 : t_2$ and $s_2 : t_2$ carries the information that $s_3 : t_3$ then $s_1 : t_1$ carries the information that $s_3 : t_3$.

**(2)Logic as Information Flow**

$$\frac{t_1 \vdash t_2}{s : t_1 \longrightarrow s : t_2}$$

If the type $t_1$ entails $t_2$ then $s : t_1$ carries the information that $s : t_2$.

**(3)Addition of Information**

$$\frac{s_1 \ : \ t_1 \ \longrightarrow \ s_2 \ : \ t_2 \quad s_1 \ : \ t_1' \ \longrightarrow \ s_2 \ : \ t_2'}{s_1 \ : \ t_1 \wedge t_1' \ \longrightarrow \ s_2 \ : \ t_2 \wedge t_2'}$$

If $s_1 : t_1$ carries the information that $s_2 : t_2$, and $s_1 : t_1'$ carries the information that $s_2 : t_2'$ then $s_1 : (t_1 \wedge t_1')$ carries the information that $s_2 : (t_2 \wedge t_2')$.

**(4)Exhaustive Cases**

$$\frac{s_1 \ : \ t_1 \ \longrightarrow \ s_2 \ : \ t_2 \vee t_2' \quad s_2 \ : \ t_2 \ \longrightarrow \ s_3 \ : \ t_3 \quad s_2 \ : \ t_2' \ \longrightarrow \ s_3 \ : \ t_3}{s_1 \ : \ t_1 \ \longrightarrow \ s_3 \ : \ t_3}$$

**(5)Contraposition**

$$\frac{s_1 \ : \ t_1 \ \longrightarrow \ s_2 \ : \ t_2}{s_2 \ : \ \neg t_2 \ \longrightarrow \ s_1 \ : \ \neg t_1}$$

Where $\wedge$, $\vee$, $\neg$, and $\vdash$ are given a (classical) logical interpretation; e.g., $s \ : \ t_1 \wedge t_2$ iff $s : t_1$ and $s : t_2$.

### 3.3 Information Flow as Arrows

We formalize information flow using arrow logic. We define a site as a model and an end-point, a type as a proposition, and an of type relation between a site and a type as a models relation between a model, an end-point and a proposition. Then we rewrite the above five principles using the notation of arrow logic. We redefine information flow by arrow logic:

$M, \langle x, y \rangle \models \phi \rightarrow \psi$ iff $M, x \models \phi$ and $M, y \models \psi$

For example, we rewrite Xerox Principle as follows:

$$\frac{M, a \models \phi \rightarrow \psi \quad M, b \models \psi \rightarrow \theta}{\exists c \ M, c \models (\phi \rightarrow \psi) \bullet (\psi \rightarrow \theta) \text{ and } M, c \models \phi \rightarrow \theta}$$

## 4 Logic of Imperfect Information Flow

In this section, we propose a formalization of communication channels between agents, together with communicative agents. Here we mainly discuss the following three problems.

**Channel Bottleneck** A channel may not have enough capacity to let a large amount of information flow through, and in that case, we need to divide the information into their parts. However, the division by logical connectives, such as conjunction or disjunction of propositions may not be able to preserve the original meaning.

**Unreliable Channel** A channel in the real world might have a sound quality to let information through, and it may add some noise on information. We discuss the reliability of channels in terms of noises, or programs functions upon input sentences.

**Reverse Information** Reverse information can be regarded as a simple acknowledgment, so that the receiver returns the same information to the sender, or otherwise the receiver returns an asking-back for uncertainty. Otherwise, it may be a notification of channel error. In general, we consider that a sender who sent information receives its reverse information.

Our formalization of a channel is depicted as in Fig. 3. An agent utters a sentence the content of which is a proposition $\Phi$ to the other agent. A receiver perceives it, however, the propositional content changes to $\Phi^c$, where the modal operator '$c$' is put on $\Phi$, because the identity of $\Phi$ is not ensured if the channel is unreliable. The utterance and the perception together are regarded as a communication channel between these agents. This kind of a channel through which information flows can be considered to be a program, in the sense that a sentence is an input and the perceived information is an output, where noises can be added indeterministically. This channel is an unreliable channel. Here, we formalize the noise to be some modal operators.

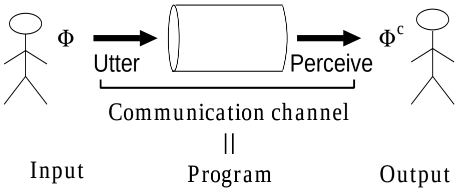

**Figure 3:** Proposed model of communication channel

### 4.1 Definition

When we regard that information flow is communication between agents , how should we locate the modal operators in the diagram? We can consider the following three alternatives, where $O$ is some operator.

1. An operator $O$ is a **site**. Namely, $O_a\Phi$ can be implemented as $O_a \models_S \Phi$ in a site.
2. An operator $O$ is a **channel**. Namely, $O_a\Phi$ can be implemented as $\Phi \overset{O}{\Rightarrow} \Phi$.
3. An operator is just a **part of a sentence**.

where $s \models_S \sigma$ means that the infon $\sigma$ is valid in the situation $s$, or in other words, $s$ supports $\sigma$. We call such a form as: $\ll P, a_1, a_2, \cdots, a_n, 1/0 \gg$ an *infon*, meaning that objects $a_1, a_2, \cdots, a_n$ have the relation $P$. A situation $s$ is a set of infons $\{\sigma \mid s \models_S \sigma\}$. $\Phi, \Psi, \cdots$ are natural language sentences. (Note that $\models_S$ is a support relation, and is different from a model $\models$).

From now on, we formalize a communication channel between agents in terms of information flow. We interpret the language of the above information flow into that of arrow logic, as follows.

| Information Flow | Arrow Logic |
|---|---|
| a site ($\mathcal{K}_a$) | a model and an end-point |
| a type | a proposition |
| a support relation $\models_S$ | a model relation $\models$ |
| (between a site and an infon) | (between a model, an end-point and a proposition ) |

### 4.1.1 $K$ as Site

We may use an agent name directly as a site name, however, we formalize the site as the knowledge of the agent, as is $K_a$ by Hinttikka [Hintikka 1962]. Thus, if we are to model communicative agents and the information flow between them, the knowledge of each agent should be the information site. A natural language sentence $K_a\Phi$ becomes a proposition: $\mathcal{K}_a \models_S \Phi$. (We use calligraphic characters for site names, and distinguish them from operators.) This proposition intuitively means that "the agent $a$ have information $\Phi$ in her knowledge $\mathcal{K}_a$."

### 4.1.2 '$c$' as Channel

We define that $\Phi^c$ is such a proposition as $\Phi$ connected with the operator '$c$', meaning that $\Phi$ is changed by some noise, and usually $\Phi^c$ cannot be separated into $\Phi$ and '$c$'. We regard the operator '$c$' as a channel. An agent's dispatch of information is to put the information on the other agent's perception channel. We can locate $K$'s and '$c$' as:

$$\mathcal{K}_a \models_S \Phi \longrightarrow \mathcal{K}_b \models_S \Phi^c,$$

where the agent $a$ knows $\Phi$ and utters $\Phi$, then the agent $b$ perceives $\Phi^c$.

If the channel is reliable, '$c$' is regarded as the perception $P$ defined in section1, so $\Phi^c$ implies $\Phi$ (1). If the channel is unreliable and $\Phi$ should be reconstructed by some method from $\Phi^c$, $\Phi^c$ implies $\Diamond\Phi$. Then if the agent believes $\Phi$, $B\Phi$ is in the agent's knowledge (2).

$$\mathcal{K}_a \models_S \Phi \longrightarrow \mathcal{K}_b \models_S \Phi^c \supset \Phi \tag{1}$$
$$\mathcal{K}_a \models_S \Phi \longrightarrow \mathcal{K}_b \models_S \Phi^c \supset B\Phi \tag{2}$$

### 4.1.3 $B$ as Part of Sentence

We define the other remaining operator, the belief operator $B$ as a part of sentences. Thus, $\mathcal{K}_a \models_S B_a\Phi$ means that "the agent $a$ believes $\Phi$ in site $\mathcal{K}_a$."

## 4.2 Agent Communication through Channels

In this section, we discuss several features of agent communication as information flow, to verify the applicability of our model.

### 4.2.1 The Application of our Model to the Test Principles

We inspect whether these principles are valid depending upon the reliability of the channel.

**(1)Xerox Principle**

- reliable channel: valid

$$\frac{\mathcal{K}_a \models_S \varPhi \longrightarrow \mathcal{K}_b \models_S (\varPhi^c \supset)\varPhi \quad \mathcal{K}_b \models_S \varPhi \longrightarrow \mathcal{K}_c \models_S (\varPhi^c \supset)\varPhi}{\mathcal{K}_a \models_S \varPhi \longrightarrow \mathcal{K}_c \models_S (\varPhi^c \supset)\varPhi}$$

- unreliable channel: invalid

$$\frac{\mathcal{K}_a \models_S \varPhi \longrightarrow \mathcal{K}_b \models_S (\varPhi^c \supset)B\varPhi \quad \mathcal{K}_b \models_S B\varPhi \longrightarrow \mathcal{K}_c \models_S ((B\varPhi)^c \supset)BB\varPhi}{\mathcal{K}_a \models_S \varPhi \longrightarrow \mathcal{K}_c \models_S (\varPhi^c \supset)B\varPhi}$$

This principle means that if the channels are reliable and transparent, the composition of more than two channels is reliable and transparent, but if the channels are unreliable and opaque, the composition is unreliable and opaque.

**(2)Logic as Information Flow**

- reliable channel: valid

$$\frac{\varPhi \vdash \varPsi}{\mathcal{K}_a \models_S \varPhi \longrightarrow \mathcal{K}_a \models_S (\varPhi \vdash)\varPsi}$$

- unreliable channel: invalid

$$\frac{\varPhi \vdash \varPsi}{\mathcal{K}_a \models_S \varPhi \longrightarrow \mathcal{K}_a \models_S B\varPhi}$$

The result of the applications shows that the identity channel have to be reliable and transparency.

**(3)Addition of Information**

- reliable channel: valid

$$\frac{\mathcal{K}_a \models_S \varPhi \longrightarrow \mathcal{K}_b \models_S (\varPhi^c \supset)\varPhi \quad \mathcal{K}_a \models_S \varPsi \longrightarrow \mathcal{K}_b \models_S (\varPsi^c \supset)\varPsi}{\mathcal{K}_a \models_S \varPhi \wedge \varPsi \longrightarrow \mathcal{K}_b \models_S \varPhi \wedge \varPsi}$$

- unreliable channel: valid

$$\frac{\mathcal{K}_a \models_S \varPhi \longrightarrow \mathcal{K}_b \models_S B\varPhi \quad \mathcal{K}_a \models_S \varPsi \longrightarrow \mathcal{K}_b \models_S \varPsi}{\mathcal{K}_a \models_S \varPhi \wedge \varPsi \longrightarrow \mathcal{K}_b \models_S B(\varPhi \wedge \varPsi)}$$

The information through only one unreliable channel is the same as the conjunctive information through two parallel unreliable channels.

**(4)Exhaustive Cases**

- reliable channel: valid

$$\frac{\mathcal{K}_a \models_S \varPhi \vee \varPsi \rightarrow \mathcal{K}_b \models_S \varPhi \vee \varPhi' \quad \mathcal{K}_b \models_S \varPhi \rightarrow \mathcal{K}_c \models_S \varPsi \quad \mathcal{K}_b \models_S \varPhi' \rightarrow \mathcal{K}_c \models_S \varPsi}{\mathcal{K}_a \models_S \varPhi \vee \varPsi \rightarrow \mathcal{K}_c \models_S \varPsi}$$

- unreliable channel: invalid

$$\frac{\mathcal{K}_a \models_S \varPhi \vee \varPsi \rightarrow \mathcal{K}_b \models_S B(\varPhi \vee \varPsi) \quad \mathcal{K}_b \models_S B\varPhi \rightarrow \mathcal{K}_c \models_S BB\varPhi \quad \mathcal{K}_b \models_S B\varPsi \rightarrow \mathcal{K}_c \models_S BB\varPsi}{\mathcal{K}_a \models_S \varPhi \vee \varPsi \rightarrow \mathcal{K}_c \models_S B\varPsi}$$

Remembering the logical features of belief operator in the introduction, we show the following one:

$$B(\Phi \vee \Psi) \mathrel{\overset{\not\supset}{\underset{\not\subset}{}}} B\Phi \vee B\Psi$$

Therefore,

$$\mathcal{K}_b \models_S \Phi \vee \Psi \not\rightarrow \mathcal{K}_b \models_S \Phi \text{ or } \mathcal{K}_b \models_S \Psi$$

**(5)Contraposition**

- reliable and unreliable channels: invalid
  We cannot consider the inverse channel usually. However, we can find the examples of the reverse information. We discuss it in 4.2.5.

Xerox principle and the addition of information principle are especially important in the five principles for the following reasons. When we adopt arrow logic in our formalization, as in the previous section, the first Xerox principle can be regarded as arrow composition. The second principle concerns multiple channels between agents, and we can consider the problem of *channel bifurcation* and *channel confluence*. In the following subsection, we take a look into the problem of channel bottleneck as a variant of channel bifurcation/confluence, that we often encounter in the real world.

### 4.2.2 Unreliable Channel

We define a communication channel between agents as follows. We regard a channel as an arrow and an agent as an end-point of the arrow.

$M, a \models C\Phi$ iff $M, a \models L\Phi$ and $M, a \models R(\Phi^c)$

This definition is redefine with the end-point representation:

$M, \langle x, y\rangle \models C\Phi$ iff $M, x \models \Phi$ and $M, y \models \Phi^c$

If the channel is reliable, '$c$' is regarded as the perception $P$ as was defined in section1, so $\Phi^c$ implies $\Phi$. If the channel is unreliable and $\Phi$ is reconstructed by some method from $\Phi^c$, $\Phi^c$ implies $\Diamond\Phi$. Then if the agent believes $\Phi$, $B\Phi$ is in the agent's knowledge. Accordingly, we can define a reliable channel $C_r$ and an unreliable channel $C_u$, as follows:

$M, \langle x, y\rangle \models C_r\Phi$ iff $M, x \models \Phi$ and $M, y \models \Phi$

$M, \langle x, y\rangle \models C_u\Phi$ iff $M, x \models \Phi$ and $M, y \models B\Phi$

However, it means that there already exist two end-points $x, y$ that satisfy the above conditions and the arrow $a = \langle x, y\rangle$ can be defined. Thus, if there exists an arrow $a$ and its left end-point is $x$ while right end-point is unknown, and the end-point $x$ satisfies $M, x \models \Phi$ and the arrow $a$ satisfies $M, a \models C\Phi$, then, there exists a end-point $y$ satisfies $M, y \models \Phi$.

Examples of reliable channel are quite common in human communication. Suppose that $a$ utters a sentence to tell the other agent $b$, and in that case, $b$ often misunderstands the propositional contents of what $a$ utters. Thus, the unreliability is the inherent feature of natural language. Or, let us consider the case of robot communication. In this case, the channel of visual perception is less reliable than communication channel because visual perception is more difficult technology than verbal communication.

### 4.2.3 Channel Bottleneck

If the channel does not have enough capacity, $B$ sentences is decomposed to sub-sentences (see the following formula (3)), although those sub-sentences can be transmitted via channels independently (4) or reliable channels, they can never be retrieved as the sender's original meaning ((7),(8) or (11) in reliable channel).

If the arrow(channel) $a$ satisfies $M, a \models C\phi$, then;

$$\begin{aligned}
M, a \models L\ B(\phi \wedge \psi) &\to M, a \models L(B\phi \wedge B\psi) && (3)\\
&\to M, a \models LB\phi \text{ and } M, a \models LB\psi && (4)\\
&\to M, a \models R(B\phi)^c \text{ and } M, a \models R(B\psi)^c && (5)\\
&\to M, a \models R((B\phi)^c \wedge (B\psi)^c) && (6)\\
&\not\to M, a \models R((B\phi \wedge B\psi)^c) && (7)\\
&\not\to M, a \models R(B(\phi \wedge \psi))^c && (8)
\end{aligned}$$

If the channel is reliable,

$$\begin{aligned}
M, a \models L\ B(\phi \wedge \psi) &\to M, a \models L(B\phi \wedge B\psi) &&\\
&\to M, a \models LB\phi \text{ and } M, a \models LB\psi &&\\
&\to M, a \models R(B\phi)^c \text{ and } M, a \models R(B\psi)^c &&\\
&\to M, a \models RB\phi \text{ and } M, a \models RB\psi && (9)\\
&\to M, a \models R(B\phi \wedge B\psi) && (10)\\
&\not\to M, a \models RB(\phi \wedge \psi) && (11)
\end{aligned}$$

We explain the channel bottleneck through the robot navigation example [see Fig.4]. If the channel has enough capacity and the sender sends a command '(1)Turn right and (2)Pick up •', the receiver does '(1)and(2)' or does not do '(1)and(2)'. However, if the channel has not enough capacity, the sender decomposes a command '(1)and(2)' to subsentences '(1)' and '(2)' and sends independently via channels. Then the receiver receives them and acts. The receiver does one of the following actions: (a)Turn right and pickup •, (b)Pickup and turn right, (c)Pickup only, (d)Turn right only, and do nothing. This is bad because the receiver does the action by the commands incompletely.

### 4.2.4 Iteration of Unreliable Channel

In this paper the point we wish to stress is that using arrow logic we can distinctly deal with the communication with some modal operators like belief between agents. Therefore we can more clearly analyze the behavior of multi-agents system. Then, we define the iteration of this unreliable channel $C_u$:

$M, \langle x, y\rangle \models C_u^* \phi$ iff $\langle x, y\rangle$ *can be C-decomposed into finite sequences*
$\langle x, x_1\rangle, \langle x_1, x_2\rangle, \cdots, \langle x_i, y\rangle$ *of arrows satisfying*
$M, x \models \phi$, $M, x_0 \models B\phi$, $M, x_1 \models BB\phi$,
$\cdots$, $M, x_i \models \underbrace{BB\cdots B}_{i+1}\phi$, $M, y \models \underbrace{BB\cdots B}_{i+2}\phi$

This satisfies the following principles:

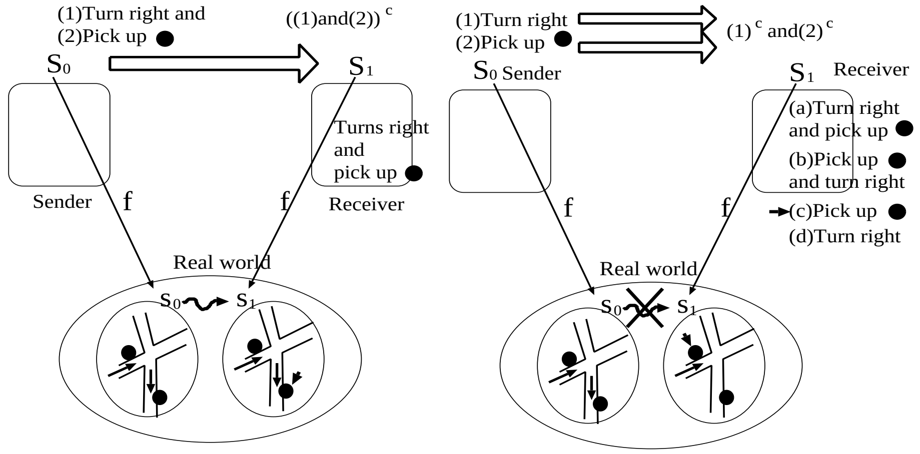

**Figure 4:** Channel bottleneck

*Axiom* $C_u\phi \rightarrow C_u^*\phi$
*Axiom* $C_u^*\phi \bullet C_u^*\phi \rightarrow C_u^*\phi$
*Rule* *if* $C_u\phi \rightarrow \alpha$ *and* $\alpha \bullet \alpha \rightarrow \alpha$ *are provable, then*
*so is* $C_u^*\phi \rightarrow \alpha$

So we can prove the following channel bottleneck proposition about unreliable channel $C_u^*$:

$$C_u^*(\phi \wedge \psi) \mathrel{\substack{\rightarrow \\ \nleftarrow}} C_u^*\phi \wedge C_u^*\psi$$

$$C_u^*(\phi \vee \psi) \mathrel{\substack{\nrightarrow \\ \nleftarrow}} C_u^*\phi \vee C_u^*\psi$$

### 4.2.5 Reverse Information

We can find a lot of examples of reverse information. Reverse information can be regarded as a simple acknowledgment that a receiver returns the same as received information to the sender, an asking-back which is repeated after a sender for uncertain information, or a notification of channel error. We consider that a sender who sent information receives its reverse information. A composition of sent information and reverse information is identity information. The following proposition (12) shows it. It means that it has no content. However the situation that reverse information returns is meaningful.

$$\Phi \bullet \Phi^{\vee} \rightarrow Id \tag{12}$$

In the robot navigation, the sender's observation can be regarded as the reverse information [see Fig.5].

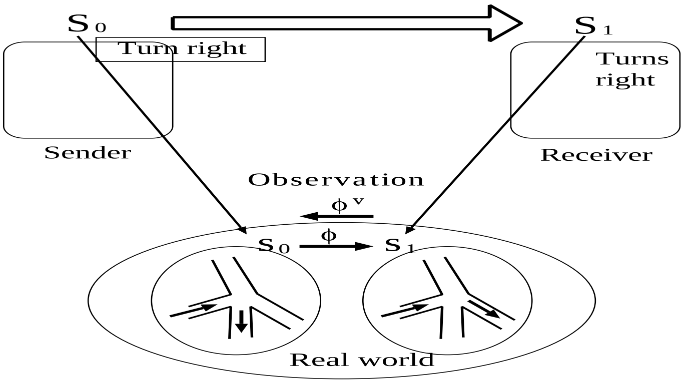

**Figure 5:** Reverse information

## 5 Discussion

In this paper, first, we show the problems of agent communication from the human language communication and the robot language communication. Thereafter, we formalized information flow using arrow logic and show Barwise's five test principles can be rewritten by arrow logic. Then we propose a formalization of communication channels between agents, as well as that of communicative agents, and we show the following three features of the communication channels between agents: (1)channel bottleneck, (2)unreliable channel, and (3)reverse information.

Finally, we consider the further possibility of dynamic arrow logic in analysis of imperfect information flow.

The first issue is the problem of channel order. If we were to communicate each other, splicing multiple pipes consecutively, we would be required to connect them properly. Namely, Xerox principle is valid if two channels are connected in a right order. $C^3x, yz$ does not necessarily imply $C^3x, zy$, and thus, $B_yB_z\phi$ may be meaningful information while $B_zB_y\phi$ may be a junk. We can consider this kind of order problem from the viewpoint of arrow logic.

The dynamic logic offers a duality between edges and vertices, viz., a site may be expanded into an arrow and an arrow may be compressed into a site. Actually, what we have done and are trying to do is the modeling of agents; in some phases, it is convenient to represent an entangled arrow network as a site, and in other phases, it is required that an internal structure of a site should be decomposed. Thus, we believe that dynamic arrow logic can be a proper tool for agent model.

This paper mainly contributes to the formalization of communication with knowledge, belief, and perception. In addition, the relation between belief and uncertain communication channel discussed in the previous section could be applied to the remodeling of multi-agent environment, for example, to control robot behavior, and so on. We think the formalization of multi-agent network or channel network is useful for recovering from the noisy information contents through unreliable channels.

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