Journal of Universal Computer Science, vol. 26, no. 5 (2020), 604-623
submitted: 26/6/19, accepted: 27/5/20, appeared: 28/5/20

Knowledge Geometry in Phenomenon Perception and Artificial Intelligence

João Gabriel Lopes de Oliveira
(Federal University of Rio de Janeiro, Rio de Janeiro, Brazil
joaogabriellopes@poli.ufrj.br)

Pedro Moreira Menezes da Costa
(Federal University of Rio de Janeiro, Rio de Janeiro, Brazil
pedromoreiramcosta@poli.ufrj.br)

Flávio Luis de Mello
(Federal University of Rio de Janeiro, Rio de Janeiro, Brazil
fmello@poli.ufrj.br)

Abstract: Artificial Intelligence (AI) pervades industry, entertainment, transportation, finance, and health. It seems to be in a kind of golden age, but today AI is based on the strength of techniques that bear little relation to the thought mechanism. Contemporary techniques of machine learning, deep learning and case-based reasoning seem to be occupied with delivering functional and optimized solutions, leaving aside the core reasons of why such solutions work. This paper, in turn, proposes a theoretical study of perception, a key issue for knowledge acquisition and intelligence construction. Its main concern is the formal representation of a perceived phenomenon by a casual observer and its relationship with machine intelligence. This work is based on recently proposed geometric theory, and represents an approach that is able to describe the influence of scope, development paradigms, matching process and ground truth on phenomenon perception. As a result, it enumerates the perception variables and describes the implications for AI.

Key Words: knowledge geometry, perceived phenomenon, artificial intelligence, projection system
Category: M.4, M.0, F.1.1

1 Introduction

Knowledge reasoning and machine learning are two distinct approaches for addressing the knowledge acquisition problem. Shoham [Shoham, 2016] points out that in the 1990s artificial intelligence (AI) focused on logic-based knowledge, but today focuses on machine learning, and argues that this shift served AI well, but that a backward shift is necessary. Although it seems reasonable that a better balance between the two approaches might be useful, and that both are encompassed by the knowledge acquisition problem, curiously the modeling of the phenomena to be known remains a bottleneck and poorly studied. The main concern of this article is the formal representation of such phenomena and its relationship with machine intelligence. It describes the advances in this ongoing theoretical research, sometimes using philosophical proofs with analytical reasoned arguments, and others using mathematical proofs based on euclidean geometry.

Human intelligence arises from an amazing duality, reaching conclusions based on the perception of aleatory patterns and on very structured and rational decisions. Both forms are distinct and complementary. Machine intelligence, in turn, also arises in two ways: (i) machine learning, which interprets patterns in data to arrive at conclusions and hence mimics, roughly, perception by the human brain; and (ii) reasoning systems, which use standard logic and chained deductions in order to mimic the rational intelligence of the human mind. The boundary between a theory of human thinking and a scheme for making an intelligent machine is not clear, assuming that it does indeed exist. From this point of view, theories of AI are in fact theories of mind.

Perception plays an important role in AI since it processes environment features captured by sensors and provides useful data for planning and acting upon that environment thought actuators. There are different ways in which knowledge acquisition can be made operational, either by humans or by machines, but the basic source of knowledge is the process of perception [Alston, 2017][Pillow, 1989]. The end product of this process is a representation of the perceived phenomenon [Lieto et al., 2017]. Although such representations have been the object of much study and debate within the field of AI, we claim herein that phenomenon perception has been underestimated.

Therefore, this work proposes a mathematical model for such perceived phenomenon and discusses its consequences for AI. In the following section we highlight works that have somehow influenced this research, followed by a shortened review on Knowledge Geometry Theory [Mello and Carvalho, 2015]. In section 4, we provide the first new contribution of this work, that is, a formal description of perceived phenomena using this theory. In the subsequent section we make a new study on the mathematical representation of a generic phenomenon according to AI computational models. Finally, in section 6, we describe a brand new analysis of the implications for Artificial Intelligence, and then close with a brief conclusion.

2 Related Work

Knowledge representation plays five distinct roles: as a surrogate, as a set of ontological commitments, as a fragmentary theory of intelligent reasoning, as a medium for pragmatically efficient computation, and as a medium of human expression [Patel and Jain, 2018][Smith and Eckroth, 2017][Davis et al., 1993]. So, it is important not only to look for models describing these roles, but also create a consistent formalism, which is the main concern of this article.

Gärdenfors highly influential work on conceptual spaces [Gärdenfors, 2004] addresses areas of AI that are directly dependent on the knowledge representation problem: reasoning systems and machine learning. He describes possible approaches despite being skeptical about the existence of convincing evidence that these responses will succeed. Compton et al. [Martnez-BZjar et al., 2001] and Richards [Richards, 2003] referred to ripple-down rules, whereby users provide feedback to expert systems as they are used, and ontologies for explicit specification of conceptualization. Lewis and Lawry [Lewis and Lawry, 2016] used conceptual spaces and random set theory to introduce a hierarchical framework for combining conjunctive concepts. Conceptual spaces were used to represent correlations between different domains [Bechberge and Kühnberger, 2017]. Moreover, Gärdenfors [Gärdenfors, 2014] proposed that the meanings of words can be described in terms of geometric structures. This author [Gärdenfors, 2000] also introduced topology into concept structure, for which two properties of regions are desired: connectedness, whereby regions cannot be decomposed into smaller regions without those regions intersecting each other; and convexity, whereby every line connecting two points passes only through the region. Although the present article does not follow the same approach as Gärdenfors, this author's work encourages the use of geometry and topology as potential tools for a formal description of concepts.

Grenander [Grenander, 1997] discusses a formal representation of knowledge called Geometries of Knowledge. A computer is unforgiving when it comes to nebulous or ambiguous instructions, and thus we are forced to formalize the understanding of the research object in well-defined logical categories so that they can be correctly translated into computer code. Grenander [Grenander, 1997] studies patterns of theoretical representations of knowledge with the propose of automating components of human mental activities, such as recognition. This leads the discussion towards machine vision, but the main idea of using geometry for representation presented by this article stand still. Stroing [Ströing, 2018] proposed a connection between Grenanders work, phenomena and data patterns. Independently from the present work, Ströing also noticed that the notion of phenomenon is a key issue for further developments in AI.

The process of making sense of complex data at an abstract and conceptual level is fundamental to human cognition. Chalmers et al. [Chalmers et al., 1992] called this high-level perception; that is, mental representations are used throughout cognitive processing in order to organize chaotic environmental stimuli. Such research goes deeper into what perception is and what its consequences are for AI. Laird et al. [Laird et al., 2017] also discussed cognitive architectures as a way to provide appropriate computational abstraction for defining a standard model of mind. Lieto et al. [Lieto et al., 2018] made it clear that abstract models of cognition and the software instantiations of such models employed in AI are tightly coupled. These papers are particularly relevant to the present work because they try to model phenomena perceived by a computationally intelligent system.

3 Abridged Review on Knowledge Geometry

It is important to recall the fundamental concepts of knowledge geometry for the purpose of introducing the reader to this theory. This also allows projection operators to be understood, which are a main issue of this paper. While this brief summary provides the information necessary to support the scope of this article, the reader is encouraged to consult the original publication for a broader discussion [Mello and Carvalho, 2015].

The observation of reality through sense organs allows us to acquire some understanding of things that enables the elaboration of concepts of the outside world. Knowledge, therefore, is based on an acculturation process. Besides sensorial observation, other types of perception and, in particular, the human minds deductive processes, contribute to the formation of concepts. Hofstadter [Hofstadter, 1985] introduced the concept of implicosphere by defining it as a cloud of impressions about an object, a class of objects, a relationship among objects or a phenomenon. This cloud (Figure 1(a)) becomes thicker and more complex as a casual observer experiences deeper contact with such entities. The implicosphere contains what we know about an object or phenomenon, and thus is a theory about the object (phenomenon). In Knowledge Geometry Theory, the vector space containing a phenomenon is the so-called real plane, which is a simplification since it forces a bi-dimensional representation, but it does not lack generality. A phenomenon has additional dimensions, but the geometry and operations of this theory can be adapted to support these extra dimensions, meaning this increment is viable but not essential.

Objects, classes, relations or states observed through implicospheres are projected on to what is termed the conceptual plane. The corresponding theories are projected on to another plane called the symbolic plane, where the concepts are registered as symbols (Figure 1(a)). What can be noticed in the real plane by directly using our sense organs differs from what can be observed using instruments. Indeed, there is a diversity of real planes, which approximate ever closer the conceptual plane up to the point that they may even become mixed, since perception of reality is largely a consequence of an acculturation process, and thus is highly induced by the knowledge or understanding one has at the moment of observation.

Figure 1(b) presents the conceptual plane as a filter for perceptions of reality. Different observers have different filters, and the accumulation of observations modifies the effect of the performance of the cultural filter for a given observer.

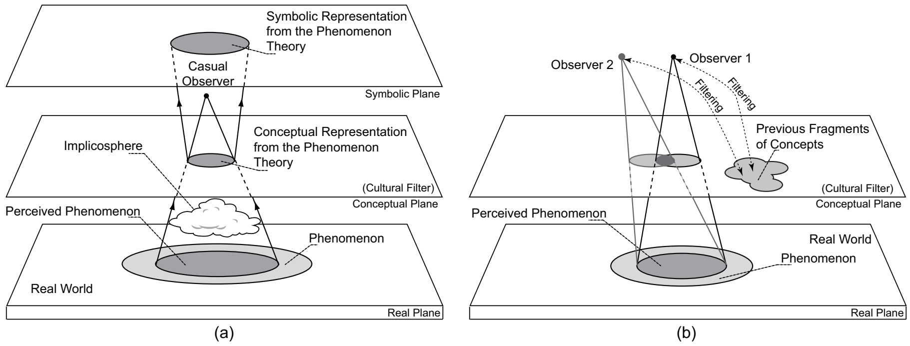

Figure 1: Real, conceptual and symbolic planes: (a) implicosphere for a casual observer; (b) different observers and different concepts.

The cultural filter is an evaluation system for our perceptions that enables the generation of concepts. These concepts, in turn, will influence the evaluation process by modifying the cultural filter through which reality is perceived.

The perceived concept is the result of the observers casual observation, which is influenced by their cultural filter. In turn, the abstracted concept results from a focused and intentional view, which reveals the observers interest in the phenomenon. Therefore, it is supposed that only a central part of a concept is registered, while the rest is ignored. Abstraction, thus, is an abbreviating of what was observed; the central region, which corresponds to the abstracted concept, remains.

Concepts obtained through abstraction modify the cultural filter and so, from further observation of reality, one perceives or imagines that they perceive new objects or phenomena that were not previously observed. This is the projection process called reification of abstractions. Reification, thus, amplifies the understanding of the perceived phenomenon and, in the upper bound case, it may occasionally include the real phenomenon itself. The next step is to redefine once again the perceived concept through discernment, still independent from reasoning or analysis. From this viewpoint, reification modifies the previously formed concept. Such a change is called inference by intuition.

There is another way to perfect concepts, without using the real plane to perform projections and back projections, which employs the process of abstraction formalization on the symbolic plane. Such formalization targets the understanding of concepts according to their formal structures through algebraic symbols and axioms operated by means of well-defined syntactic rules. Axioms obtained through formalization can now be interpreted by modifying the concepts used for formalization, thus enriching understanding. This interpretation is the process of determining the precise understanding of a concept by using induction; that is, by applying reasoning from particular data combined with cognitive operations to reach more general concepts or, in other words, conceptual consequences. The symbolic knowledge acquired through such projection is referred to as logical consequences. This process is deductive, a logical inference of reasoning. The result is the production of logical consequences, such as those achieved in Post's Production Systems and Chomsky's Grammars.

4 Novel Study on Geometrical Representation of Phenomenon Perception

Since knowledge geometry is based on intersections of cones and planes, it is necessary to develop a study on the relationships between the two shapes in the context of this theory, which is essential to further discussions on the limits of perception for humans and machines. This section, and the following ones, describe new facts and contributions to the formal representation of such perception phenomena and its relationship with machine intelligence.

Conic views are used in the proposed theory because perspective projections deform reality. Let's start this study from Figure 2 that illustrates the vision of a casual observer O looking at some phenomenon in the real plane through a field of view (aperture) α. The z-axis of the coordinate system is defined as coincident to the axis of the cone, and thus passes through the position of the observer $O(x_0, y_0, z_0)$ and is orthogonal to the plane (λ) where the base of the cone lies. An oblique cone is a more generic representation, but it would make the calculus described below much more extensive without making an important contribution to the model, and so in this paper we use an orthogonal cone. Moreover, Figure 2 does not illustrate the upper nappe of the cone, but this issue will be analyzed soon. The directrix lies on the plane defined by the other axis of the coordinate system and describes an ellipse, with axes a and b, located at a distance of c from the observer O. Therefore, the generic cone vision equation and the equations for the planes are given by:

$$\begin{aligned}
\chi &: \frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = \frac{(z-z_0)^2}{c^2} \\
\pi &: \mathbf{n} \cdot \overrightarrow{FP} = 0 \\
\lambda &: \mathbf{m} \cdot \overrightarrow{GP} = 0
\end{aligned}$$

The cone vision can be simplified to a circular cone with radius r, that is, $a = b = r$. This assumption simplifies the calculation without significantly decreasing generality, so $\tan \alpha/2 = r/c$. Let the intersection of cone χ with plane λ be the circle ω. Note that there is a convenient reference system, which allows

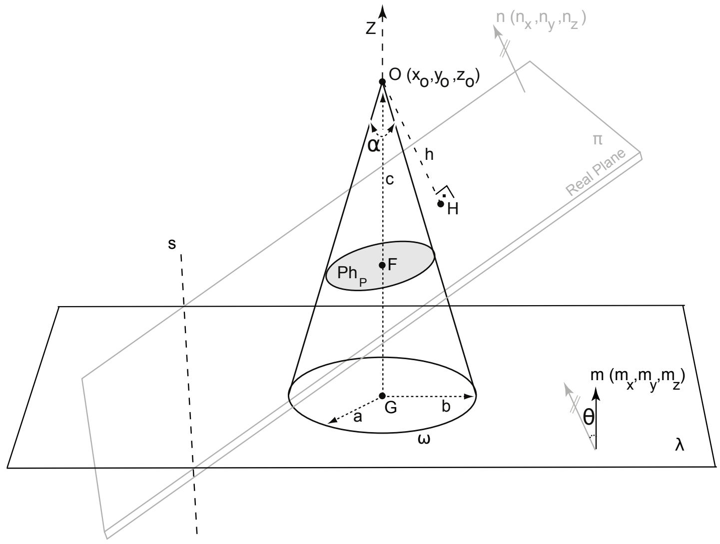

Figure 2: Cone vision and real plane.

for less extensive representations, based on the Cartesian coordinate system, where $\mathbf{m} = (0, 0, 1)$ and $G = (0, 0, 0)$. So:

$$
\begin{aligned}
\omega :& x^2 + y^2 = r^2 \\
& x^2 + y^2 = c^2 \tan^2 \alpha/2 \\
\pi :& n_x x + n_y y + n_z z - n_x x_F - n_y y_F - n_z z_F = 0 \\
\lambda :& z = 0
\end{aligned}
$$

The angle between plane π and plane λ is given by θ, with the intersection (assuming they are not parallel) of these planes defining line s. Hence:

$$s : n_x x + n_y y - n_x x_F - n_y y_F - n_z z_F = 0$$

Notice that the intersection between the real plane π and the viewing cone χ is the object of study. However, before proceeding with intersection geometry, it is important to state the first theorem that establishes and relation between the phenomenon perception and the phenomenon itself.

Proposition 1. *The phenomenon perception corresponds to a percipuum, while the phenomenon itself cannot be strictly modeled.*

Proof. The reader must be aware that knowledge geometry imports concepts from Pierces Theory of Signs. Pierce divided phenomenon into two distinct objects: the percept and the percipuum [Pierce, 1958]. The former is the object independent from our minds, and corresponds to the element that presents itself to our senses. The latter is the percept as it presents itself just after perception judgment. Moreover, Pierce explains that there is no straight line between perception judgment and abduction inferences [Pierce, 1935]. Perception judgment comprises mental mechanisms that are completely beyond our control, and is insistent, compulsive and a permanent obstacle that imposes its acknowledgment. Abduction inferences are gentler, can be criticized and can even involve rules and mental training for better development. Therefore, back to Figure 1(a), the percept is the phenomenon itself, the percipuum is the immediate and unconsciously perceived phenomenon, the perception judgment is responsible for reducing the phenomenon into the perceived phenomenon, while all other projection mechanisms are abduction process. Pierce also concludes that nothing can be asserted about the phenomenon itself, except by the mediation of perception judgment, which results in the percipuum.
Moreover, this statement was called a proposition, not a theorem, because it was not proved using rigorous mathematical reasoning. However, the authors claim that the philosophical arguments presented here are evidences that allows to assume this statement to be true. □

It is interesting that there have been many semiotic studies concerning the percipuum but, to the best of our knowledge, there have been no computer science studies on this concept. Study of the percipuum may improve the understanding of perception judgment, which is essential to machine intelligence and that is why we are so interested in this immediate and unconsciously perceived phenomenon (Ph_P). Therefore, the Ph_P is the only object that can be evaluated, and corresponds to the intersection between the real plane and the cone vision. The Ph_P boundary corresponds to a curve (conic section) generated by the intersection of a plane with one or two nappes of a cone. Hence, this conic section is obtained by intersecting π and χ, which means it encompasses points that are in both π and χ. This leads to the next theorem.

Theorem 2. *Perceived phenomenon (Ph_P) may be bounded by a conic curve in the form of as a circle, ellipse, parabola and hyperbola.*

Proof. Consider a locus γ constrained by the line $s \in \pi$ and the circle ω within one of the situations illustrated in Figure 3.

When Ph_P is an ellipse, there is either one point of intersection between s and ω or no such point. When Ph_P is a circle, π and λ are parallel, and there is no line s because there is no intersection. However, when Ph_P is a parabola or a hyperbola, there are two points of intersection. The solution of system γ

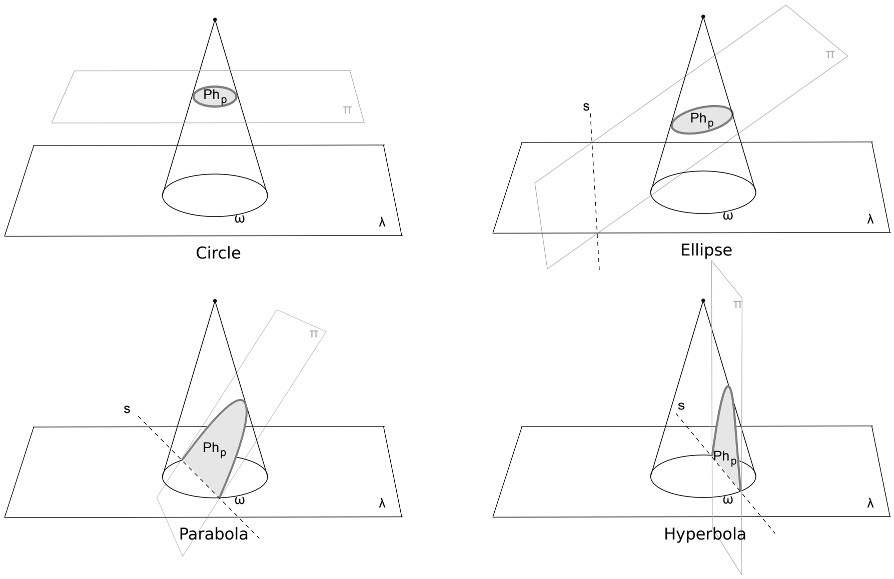

Figure 3: Types of conic sections between real plane π and observer's cone vision λ.

allows distinguishing the circumstances when Ph_P is a circle/ellipse from when it is a parabola/hyperbola, that is, whether the Ph_P region is closed and finite or open and infinite.

$$\gamma : \begin{cases} s : n_x x + n_y y - n_x x_F - n_y y_F - n_z z_F = 0 \\ \omega : x^2 + y^2 = c^2 \tan^2 \alpha/2 \end{cases}$$

$$\gamma : \left(1 + \frac{n_x^2}{n_y^2}\right) x^2 - \frac{2n_x(n_x x_F + n_y y_F + n_z z_F)}{n_y^2} x + \frac{(n_x x_F + n_y y_F + n_z z_F)^2}{n_y^2} - c^2 \tan^2 \alpha/2 = 0$$

where $n_y \neq 0$.

Therefore, γ is represented by a quadratic equation, that is, it has the form $ax^2+bx+c=0$. The discriminant $\Delta = b^2-4ac$ allows deducing some properties of the roots of this quadratic polynomial. Parabolas and hyperbolas will be associated with $\Delta > 0$, while circles and ellipses will correspond to $\Delta \leq 0$. Let $\beta = n_x x_F + n_y y_F + n_z z_F$, hence

$$\Delta = \frac{4n_x^2\beta^2 - 4(n_x^2 + n_y^2)(\beta^2 - n_y^2 c^2 \tan^2 \alpha/2)}{n_y^4}$$

Despite there being no geometrical constraint for $n_y = 0$, leading Δ to a mathematical exception, this situation is discarded by the knowledge geometry model because it would be necessary for the observer to be looking parallel to plane π. This is a useless situation because, under this circumstance, the observer would not see any phenomenon/percipuum at all, and so $n_y = 0$ does not belong to the solution space. □

Moreover, this perceived phenomenon is observed through the Hofstadter implicosphere [Hofstadter, 1985], which is an implicit counter factual sphere referring to things that never were, but that we cannot help seeing anyway. Note that, as a sphere, the implicosphere will always project convex shapes. Nevertheless, this does not mean that Ph_P regions must be convex; concave shapes are accepted as well, and the point is that such concave shapes are produced by the projection of more than one implicosphere. This kind of construction meets the studies of Gärdenfors [Gärdenfors, 2000], and subsequent literature, on the topological approach to concept structure. From this perspective, intersection and decomposition of regions are associated with the Gärdenfors' connectedness property, and natural concepts are evidenced when every line connecting two points passes only through a single region, which is the convexity property. However, we are interested on special types of regions, and the next theorem is about this.

Theorem 3. *Actual and practical perceived phenomenon (Ph_P) may be bounded only by a circle or an ellipse.*

Proof. Perception is the way of noticing things, especially with the senses, although these faculties are not restricted to natural human senses (i.e., capacities can be enhanced by artificial instruments and artifacts). This definition is still in accordance with Sartre [Sartre, 1940] as well as with his conclusion that the act of perceiving implies, as a necessary condition, spatial and temporal proximity to the object. Objects distant in time cannot be perceived and can only be evoked or imagined. Additionally, objects distant in space cannot be perceived when beyond the operational limits of receptor devices or when obstructed by barriers. A Ph_P region defined by parabolas and hyperbolas (bounds open and infinite regions) certainly trans pass operational limits of such senses and, thus, it is questionable if such phenomena are in fact perceivable. □

Do not misunderstand these formulations as a return to Pythagorism, where nature is essentially mathematical. Mathematics is exact (although its completeness and consistency is beyond the scope of this article), but nature is not (see Cartwright et al. [Cartwright et al., 2003] for an interesting discussion on the subject). This remark is particularly relevant because one might be interested in inspecting the boundary of neighboring regions, but from the practical point of view, that is, from the perspective of constructing reasoning systems and machine learning systems, this is not a pragmatical concern. When dealing with computable and workable problems, in Seldovichs words [Arnold, 2005], we are always interested only in ratios of finite increments, and never in any abstract mathematical limit.

Thus, it is possible to state that knowledge geometry concerns perceived phenomena that are simultaneously convex in shape and bounded by closed curves. These constraints leads to the next theorem.

Theorem 4. *The function prototype of a perceived phenomenon is given by a computable function* $Ph_P(\alpha, O, n, F)$*.*

Proof. If the Ph_P is simultaneously convex in shape and bounded by closed curves, then we are interested in cases where Ph_P is an ellipse (or circle), that is, when $\Delta \leq 0$. So,

$$\Delta = \frac{4n_x^2\beta^2 - 4(n_x^2 + n_y^2)(\beta^2 - n_y^2 c^2 \tan^2 \alpha/2)}{n_y^4} \leq 0$$
$$n_x^2\beta^2 - (n_x^2 + n_y^2)(\beta^2 - n_y^2 c^2 \tan^2 \alpha/2) \leq 0$$
$$(n_x^2 + n_y^2)c^2 \tan^2 \alpha/2 - \beta^2 \leq 0$$

Notice that $\overrightarrow{c} = \overrightarrow{OG}$, thus $c = |\overrightarrow{OG}| = |G - O| = |(0,0,0) - (x_O, y_O, z_O)|$, and

$$\Delta = (n_x^2 + n_y^2)(x_o^2 + y_o^2 + z_o^2) \tan^2 \alpha/2 - (n_x x_F + n_y y_F + n_z z_F)^2 \leq 0$$

So, the feasibly perceived phenomenon is defined by a function with the following dependencies:

$$Ph_P(\alpha, O, n, F)$$

where α is the field of view (aperture), O is the position of the casual observer (point of view), F is the focal point belonging to the real plane and n is the normal vector of the real plane.

Moreover, Δ value can be obtained by an effective procedure, and thus Δ is computable. Since Ph_P region is defined by a finite circumstances depending on Δ values, then Ph_P is also computable. □

In this arrangement, instead of stating that this is the condition for the conic section to be a circle or an ellipse, one can say this is the condition for knowledge acquisition (or Pierces definition of perception and Gärdenfors' concept structure).

5 Essay on Phenomenon Perception Variables

This section elaborates the meanings of the perceived phenomenon (Ph_P) variables α, n, O, F (see Theorem 4), and investigates their relationships with important concepts of AI, programming languages and computer science. The category theory abstraction [Awodey, 2010] is used to state and prove subtle mathematical results from knowledge geometry in a simple way, like the strategy proposed by Geroch [Geroch, 1985].

Let $\mathcal{C}$ be a category of sets, where its objects are theories concerning knowledge geometry, AI, programming languages and computer science, and the connections between these objects are functions, from one theory to another, that represent a processes connecting two objects. If an element is computable in one theory, it must also be computable in another theory, which is a hard assumption based on the Church-Turing Thesis. Thus, there is an isomorphism that admits a two-sided inverse, meaning that there are two morphisms in that category, $f : X \to Y$ and $g : Y \to X$, such that $gf = 1_X$ and $fg = 1_Y$, where 1_X and 1_Y are the identity morphisms of X and Y, respectively. Moreover, if $\exists x \in X$ and $\exists y \in Y$, then there is congruence between such elements, that is, $x \cong y$. Thus, we claim that objects from knowledge geometry are congruent to objects from AI, programming languages and computer science. We explain this in the following lemmas.

Lemma 1 *The aperture α is congruent to scope, that is, $\alpha \cong$ scope.*

Aperture α defines the range, or extent, of what we know about a phenomenon or, in other words, its scope. An overly reduced scope implies that one can get fewer intuitions from the phenomenon than is (theoretically) possible. On the other hand, an overly exaggerated scope can bring undesirable information with such intuitions, resulting in unnecessary computations. This scope is an object that can be found in programming languages, software engineering, AI, database design, and so on.

An important issue concerning machine learning and reasoning systems is that the construction of effective algorithms (inference, pattern recognition, classification and regression) depends on capturing discriminating, independent and informative features of the phenomenon. Webber [Webber, 2010] has previously stated that features considered important in object systems are actually simple parameters over functions and scope.

Thus, let a feature be defined as an individual measurable characteristic (property) of the object being observed. With an acute α, the observer will know less features of the phenomenon, and less attributes and aspects of the object; that is, there will be a reduced scope. An obtuse α, in turn, provides a wider scope.

Lemma 2 *The point of view O is congruent to development paradigms, that is, $O \cong$ paradigms.*

Point of view O is the way of considering the object. In fact, there are many ways of observing a given object, or many O's, since the observer may be located at any place in the space. This positioning is influenced by aspects of the cultural filter and fragments of concepts (see Section 4 and Figure 1(b)).

Wallon [Wallon, 1999] and Vygotsky [Vygotsky, 1997] discussed these aspects in their respective seminal works, and concluded that point of view is the materialization (realization) of social and ideological aspects, which are quite similar (isomorphic) to the cultural filter and fragments of concepts. These characteristics constitute a set of opinions or beliefs of a group or an individual within a certain purpose.

Moreover, it should be noted that Sommerville [Sommerville, 2011] states that software engineering is the application of tools and techniques to the development of software using a systematic methodology according to certain paradigms. The methodology is the set of rules and diligences established to conduct an activity, while the paradigm is a philosophical and theoretical framework of an engineering school within which software is developed.

Therefore, it seems that those social and ideological features manifest themselves through methods and paradigms in computer science. Thus, this is tightly coupled to software engineering and development paradigms, and any products from software development, such as those from AI. These paradigms are like *a priori* knowledge that guides analysts and developers though the task of creating a computer program.

Lemma 3 *The Real Plane's normal n is congruent to matching process, that is, $n \cong$ matching process.*

By altering the normal of the real plane it is possible to change the eccentricity of the conic that bounds the perceived phenomenon. There are two reasons for this: (1) it improves perception by creating a closed conic, as discussed in Section 4; and (2) it provides a mechanism to search for different sites and enhance phenomenon features that better sensitize one of the sensors of the observer without the penalization included in the expansion of the aperture or in the the change point of view position.

In this conceptualization process there is an attempt to get something unknown and match it with something known. Imagine a great collection of fragments of concepts stored in the cultural filter (recall Figure 1b), and one of them is evoked when evidence and expectation make it plausible that the phenomenon in view will fit it. This is much like the frame system of Minsky [Minsky, 1974], who defines a frame as a remembered structure to be adapted to fit reality by changing details as necessary, and states that collections of related frames are linked together into frame systems. Minsky reports that once a frame is proposed to represent a situation, a matching process tries to assign values to each of the frames's terminals that are consistent with the markers at each place.

Therefore, the key issue of changing the normal vector is to rotate the phenomenon (or the observer, depending of which referential system you use) and expose new features to such a matching process. Artificial intelligence and operational research use such process by employing appropriate rules to individual problem states, generate new states to which the rules can then be applied, and so forth, until a solution is found, if and only if it exists.

Lemma 4 *The focal point F is congruent to ground-truth, that is, $F \cong$ ground-truth.*

The focal point F of the phenomenon is the thing that is concentrated on or paid most attention to; that is, an objective or result towards which efforts are directed, such as a target. Let such target be the result of an unknown target function $f : X \to Y$, that is, $F = f(x)$. In machine learning, the unknown f is a function whose behavior someone wants to mimetize, and the pairs $< x_1, y_1 >, < x_2, y_2 >, \ldots, < x_n, y_n >$ are historical records of this function that are grouped into training examples. A learning algorithm then calibrates a new function g, usually called the inference model of f; that is, it constructs a new function so that $g \approx f$, where $g(x) = G \sim F$.

Note that ground-truth is a term used in various fields to refer to information provided by direct observation (i.e. empirical evidence) as opposed to information provided by inference. In AI, the key issue is to make inference coincident with such ground-truth.

The difference between G and F evokes the same difference from an evaluation system such as Delta Rule (Widrow and Hoff Learning Rule). In machine learning, network weights are updated by attempting to minimize error in the output of the neural network through gradient descent. Thus, the error for a neural network with k outputs is given by

$$E = \sum_k \frac{1}{2} (F_k - G_k)^2$$

Moreover, the distance $d(\overline{OF})$ establishes a relationship between development paradigms and ground-truth. Sometimes, this distance is so small that the concepts involved in this knowledge acquisition process are liable to be evaluated directly through observation, which are called basic concepts. On the other hand, when such a distance is too great, it may be difficult to retrieve a perception relationship between development paradigms and ground-truth. In this scenario, the capacity of noting things is compromised and needs to be enhanced by artifacts [Sartre, 1940]. The use of oracles, a knowledge reasoning mechanism, becomes relevant in this situation because it provides such artifacts, narrowing the gap between ground-truth and the maximum range the paradigm allows the observer achieve.

Theorem 5. *The Ph_P definition is congruent to another phenomenon perception definition, that takes scopes, paradigms, matching processes, ground-truths, as parameters. So,*

$$Ph_P(\alpha, O, n, F) = Ph'_P(\textit{scope, paradigm, matching process, ground-truth})$$

Proof. Since Ph_P is computable (see Theorem 4), then there is a machine M that takes α, O, n, F as inputs and return $R = Ph_P(\alpha, O, n, F)$.

If $\alpha \cong$ scope, then there is a computable function f where $f(\text{scope}) = \alpha$. Similarly there are functions such as $g(\text{paradigms}) = O$, $h(\text{matching process}) = n$ and $k(\text{ground-truth}) = F$.

Moreover, there is a machine P_1 that takes scope as input, maps f to it, and produces α. Thus, it is possible to attach the machine P_1 output to α's input at the machine M. The same procedure can be done with machines P_2, P_3, P_4 and g, h, k, respectively. This machine composition takes scope, paradigm, matching process, ground-truth as inputs and also produces R. The functional associated to this machine is called Ph'_P. □

6 Analysis of the Implications for Artificial Intelligence

The results from the previous sections allow us to bind perception variables and AI system conception. Considering that machine intelligence mimics the perceptional ability of the human brain, it comes that perception is a key issue for Artificial Intelligence.

The great effort of AI is to extract meaning from raw material by accessing concepts and making sense of situations at a conceptual level so as to, if it is possible, trigger a usually highly-interactive problem-solving process. The premise for such a task is the ability to perceive, which, according to our model, means the ability to arrange convenient values of scope, paradigm, matching process and ground-truth.

Knowledge geometry is not an unique structured projection system, but consists of many convex shape projections that are themselves connected in various ways. Each projection taken individually operates only in a micro-world of small scale or toy problems. The perceived phenomena must be of a small-scale because they become unmanageable for a single projection when they are scaled up, and the control performed by the scope variable. If it is kept smaller, this variable provides a reduced micro-world, that is, a smaller domain with less features to be concerned about, and thus provides for a better chance to produce effective AI algorithms.

Artificial intelligence has failed at attempts to find global solutions, including the most notorious - the General Problem Solver, a wide scope hypotesis. Therefore, according to our model, the task is to organize projections to operate these specific domains, with reduced scope, which can be combined together into effective larger systems or implicospheres, which can in turn be combined into higher-level concepts. In doing so, concepts emerge as a kind of collection, and it is not clear if such collections are structured or not.

We do not perceive things in a totally misguided and unconstrained sense. The paradigm variable shapes the senses, and so there is a directness in perception (some philosophers describe this as noesis). This directness stems from the cultural filter and our intention of investigating the phenomenon within a certain coherence. Hence, the universality of the statement 'all perceiving is perceiving under aspects' derives from the fact that fundamental coherence of perception experiences are grounded in the positioning of the observer, that is, spatial and cultural background positioning. This seems to be the reason for the existence of ethical and non-ethical AI. The AI product is shaped according the cultural background positioning of its creator(s).

Besides, consider two distinct prediction models created by two different machine learning algorithms which used the same dataset for training. If someone asks which is the most correct prediction model the answer must be both of them. These models are a symbolic representation of what was perceived from the phenomenon, and this perception was shaped by the paradigm that underlies the algorithm used for training. Under the viewpoint of each algorithm, each resulting prediction model is correct.

However, when someone asks which is the best model, the intention is to know which prediction model is the most desirable, which would be the model that produces inferences most coincident with the ground-truth variable. Discovering the truth-values of a proposition requires the application of semantic procedures in investigating the world. The truth of propositions, and so the truth of what is believed, is determined by the correspondence between actual or possible belief-states, thoughts or assertions and reality. But, unlike classical correspondence theory, the relationship is not required to be one-to-one in every case.

The role of ground-truth is to provide a target, a focal point of attention for the intelligent system that is being constructed. It can be a strict labeled set as in machine learning, or an objective-truth as in reasoning systems. This ground truth may be imposed or reported to the AI system, but it is not mandatory that such ground-truth be authentic. It will shape what will be learned by such a system, although the final result of this learning process may be difficult to anticipate.

There might be some circumstances where one revised output, or semi-automated process, produces the target for the next algorithm in a pipeline.

However, this algorithm revising its own targets via some rule is not exactly providing new ground-truth, it is just a clever automated refinement that is part of the model. Moreover, whenever no ground-truth is explicitly provided, like in anomaly detection and unsupervised learning, the algorithm used for learning inherent structure from the input data is biased by the paradigm. In this case, bias directs and orients the AI system toward an implicit ground-truth.

When AI systems need to investigate a world, semantic procedures evaluate the set of all admissible configurations in it. In order to search for these configurations (Minsky frames), the intelligent system must have the ability of getting something unknown and matching it with something known. Therefore, the matching process variable is also an important parameter and influences phenomenon perception. When a machine encounters a situation, it must select the memory structure, called a frame, that needs to be adapted to reality by changing details as necessary. This task will be as efficient as is the matching process.

Therefore, phenomenon perception variables (scope, development paradigms, matching process and ground-truth) help us to understand key issues of AI. Whenever an AI system conception becomes hard to design, it is possible to use the congruence of such variables with geometric structures so that the obstacles can be easily overcome using geometrical mechanisms.

7 Conclusion

The main concern of this article was the formal representation of a perceived phenomenon by a casual observer and its relationship with machine intelligence. This work suggested a mathematical model for such perceived phenomenon and its consequences for artificial intelligence. We used concepts of Knowledge Geometry Theory [Mello and Carvalho, 2015] as individual pieces for our model, and connected them with conventional geometrical rules. This work successfully organized these connections, finding that many of them represent well established features from intelligent systems. Moreover, being a geometric model, it is easier to understand the individual mechanisms involved in the process of perception and knowledge acquisition, which were enumerated and understood using familiar concepts from the areas of artificial intelligence and psychology [Mello and Souza, 2019].

As for future work, the arrangement of information throughout the real plane still needs extra attention. The arrangement needs to be made in such a way that the geometrical distance between each of the ideas represents, at least approximately, the distance of the concepts themselves. For simple cases it is trivial to manually arrange the implicosphere projection on the plane, however, more complex cases might require a more formal and established approach.

Furthermore, this work focused on the real and conceptual planes, but it is important to address and conceive a formal representation for the projection system between the conceptual and symbolic planes. It is not clear how similar theories represented on the symbolic plane are projected from the conceptual plane.

Besides, we believe that the scientific community has forgotten, or misunderstood, Hennie' [Hennie, 1977] paper on Abstract Family of Algorithms and Rogers' [Rogers Jr, 1967] Isomorphism Theorem. These are key concepts, and it seems possible to fit them into Knowledge Geometry Theory. The former talk about the algorithms indexing, and the latter uses a so called strong translation theorem. We see these ideas as useful to prove other statements derived from the present article.

Finally, we suggest a practical study to map perception problems into Knowledge Geometry. Such study may present potential application lined up with the proposing new methodology.

Acknowledgements

This project is supported by Coppetec Fundation DEL/Poli/UFRJ (Project Poli-19.257). We thank Dr. Erik Russell Wild, American biologist from University of Wisconsin, for providing a native English speaker revision of the manuscript. We would like to pay special regards to AI2Biz Lab, an Intel® AI Builder Partner, for providing an rich environment for discussions.

References

[Alston, 2017] Alston, W. (2017). *The Blackwell Guide to Epistemology*, chapter Perceptual Knowledge, pages 221–242. John Wiley & Sons.

[Arnold, 2005] Arnold, V. I. (2005). *The Role of Mathematics in Physical Sciences*, chapter Mathematics and physics, pages 225–233. Dordrecht:Springer.

[Awodey, 2010] Awodey, S. (2010). *Category Theory*. Oxford Logic Guides, 49, (2nd ed.). Oxford University Press.

[Bechberge and Kühnberger, 2017] Bechberge, L. and Kühnberger, K.-U. (2017). Measuring relations between concepts in conceptual spaces. *Artificial Intelligence XXXIV. SGAI 2017. Lecture Notes in Computer Science*, 10630.

[Cartwright et al., 2003] Cartwright, N., Psillos, S., and Chang, H. (2003). *The Modern Physical and Mathematical Sciences*, chapter Theories of scientific method models for the physico-mathematical sciences, pages 21–35. Cambridge University Press.

[Chalmers et al., 1992] Chalmers, D. J., French, R. M., and Hofstadter, D. R. (1992). High-level perception, representation, and analogy: A critique of artificial intelligence methodology. *Journal of Experimental and Theoretical Artificial Intelligence*, 4(3).

[Davis et al., 1993] Davis, R., Shrobe, H., and Szolovits, P. (1993). What is a knowledge representation? *AI Magazine*, 14(1):17–33.

[Gärdenfors, 2000] Gärdenfors, P. (2000). *Conceptual Spaces - The Geometry of Thought*. The MIT Press.

[Gärdenfors, 2004] Gärdenfors, P. (2004). Conceptual spaces as frameworks for knowledge representation. *Mind and Matter*, 2(2):9–27.

[Gärdenfors, 2014] Gärdenfors, P. (2014). *The Geometry of Meaning: Semantics Based on Conceptual Spaces*. The MIT Press.

[Geroch, 1985] Geroch, R. (1985). *Mathematical Physics*. Chicago Lectures in Physics. University of Chicago Press.

[Grenander, 1997] Grenander, U. (1997). Geometries of knowledge. *Proceedings of the National Academy of Sciences of the United States of America*, 94(n):783–789.

[Hennie, 1977] Hennie, F. C. (1977). *Introduction to Computability*. Series in Computer Science and Information Processing. Addison-Wesley.

[Hofstadter, 1985] Hofstadter, D. (1985). *Metamagical Themas: Questing for the Essence of Mind and Pattern*. Ballantine.

[Laird et al., 2017] Laird, J. E., Lebiere, C., and Rosenbloom, P. S. (2017). A standard model of the mind: Toward a common computational framework across artificial intelligence, cognitive science, neuroscience, and robotics. *AI Magazine*, 38(4):13–26.

[Lewis and Lawry, 2016] Lewis, M. and Lawry, J. (2016). Hierarchical conceptual spaces for concept combination. *Artificial Intelligence*, 237:204–227.

[Lieto et al., 2018] Lieto, A., Bhatt, M., Oltramari, A., and Vernon, D. (2018). The role of cognitive architectures in general artificial intelligence. *Cognitive Systems Research*, 48:1–3.

[Lieto et al., 2017] Lieto, A., Chella, A., and Frixione, M. (2017). Conceptual spaces for cognitive architectures: A lingua franca for different levels of representation. *Biologically Inspired Cognitive Architectures*, 19:1–9.

[Martnez-BZjar et al., 2001] Martnez-BZjar, R., Ibaez-Cruz, F., Compton, P., and Cao, T. (2001). An easy-maintenance, resuable approach for building knowledge-based systems: application to landscape assessment. *Expert Systems with Applications*, 20:153–162.

[Mello and Carvalho, 2015] Mello, F. L. and Carvalho, R. L. d. (2015). Knowledge geometry. *Journal of Information & Knowledge Management*, 14(4):1550028.

[Mello and Souza, 2019] Mello, F. L. and Souza, S. A. (2019). Psychotherapy and artificial intelligence: A proposal for alignment. *Frontiers in Psychology*, 10:263–272.

[Minsky, 1974] Minsky, M. (1974). A framework for representing knowledge. artificial intelligence memo no. 306. Technical report, AI Laboratory, Massachusettes Institute of Technology.

[Patel and Jain, 2018] Patel, A. and Jain, S. (2018). Formalisms of representing knowledge. *Procedia Computer Science*, 125:542–549.

[Pierce, 1935] Pierce, C. S. (1935). *Collected Papers (CP)*. Harvard University Press. (CP 5.181).

[Pierce, 1958] Pierce, C. S. (1958). *Collected Papers (CP)*. Harvard University Press. (CP 7.642-643).

[Pillow, 1989] Pillow, B. H. (1989). Early understanding of perception as a source of knowledge. *Journal of Experimental Child Psychology*, 47(1):116–129.

[Richards, 2003] Richards, D. (2003). Knowledge-based system explanation: The ripple-down rules alternative. *Knowledge and Information Systems*, 5(1):2–25.

[Rogers Jr, 1967] Rogers Jr, H. (1967). *Theory of Recursive Functions and Effective Computability*. McGraw-Hill Book Company.

[Sartre, 1940] Sartre, J.-P. (1940). *L'imaginaire: psychologie phénoménologique de l'imagination*. Gallimard, 15 edition.

[Shoham, 2016] Shoham, Y. (2016). Why knowledge representation matters. *Communications of the ACM*, 59(1):47–49.

[Smith and Eckroth, 2017] Smith, R. G. and Eckroth, J. (2017). Building ai applications: Yesterday, today, and tomorrow. *AI Magazine*, 38(1):6–22.

[Sommerville, 2011] Sommerville, I. (2011). *Software Engineering*. Addison-Wesley, 9 edition.

[Ströing, 2018] Ströing, P. (2018). *Scientific Phenomena and Patterns in Data*. PhD thesis, Ludwig-Maximilians University.

[Vygotsky, 1997] Vygotsky, L. S. (1997). *Mind in Society: The Development of Higher Psychological Processes*. Harvard University Press, revised edition edition.

[Wallon, 1999] Wallon, H. (1999). *De l'acte á la pensée*. Flammarion, reissue edition edition.

[Webber, 2010] Webber, A. B. (2010). *Modern Programming Languages: A Practical Introduction*. Franklin, Beedle and Associates, Inc., 2 edition.