Journal of Universal Computer Science, vol. 26, no. 10 (2020), 1331-1342
submitted: 26/3/2019, accepted: 24/10/2020, appeared: 28/10/2020

Graphic Deduction Based on Set[1]

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Abstract: Based on basic concept of symbolic logic and set theory, this paper focuses on judgments and attempts to provide a new method for the study of logic. It establishes the formal language of the extension of judgment J*, and formally describes *a*, *e*, *i*, *o* judgment, and thus gives set theory representation and graphical representation that can distinguish between universal judgments and particular judgments. According to the content of non-modal deductive reasoning in formal logic, it gives weakening theorem, strengthening theorem and a number of typical graphical representation theorem (graphic theorem), where graphic deduction is carried out. Graphic deduction will be beneficial to the research of artificial intelligence, which is closely related to judgment and deduction in logic.

Key words: formal logic, judgement, predicate, graphical representation, graphical theorem, graphical deduction
Categories: F.4, I.2.4, I.2.4, I.3.6

1 Introduction

Logic is the general science of reasoning [Russell, 1948]and the basic tool for the study of artificial intelligence. Formal logic mainly involves judgment called categorical proposition and deduction. Affirmative, negative, subject, predicate, universal, particular, singular are the key words of formal logic that can all be accurately described with predicate calculus. As the interpretation of predicate calculus depends on domain of individuals, that judgment and deduction can be shifted to predicate can broaden the application range of formal logic on the one hand and can transform the content of a non-modal judgment into the equivalent concept of a set on the other. Therefore, non-modal deduction can be deduced by Venn Diagram based on set. Moreover, computer graphics is the basis of digital image processing, and digital image processing is one of the important application areas of artificial intelligence. With the advent of deep

* This work was supported by the Fund for Major Bidding Project of National Social Science of China under Grant No. 14ZDB014 and the National Natural Science Foundation of China under Grant No. 61170322.

convolutional neural networks, image classification [Krizhevsky, Sutskever, HintonImage, 2012], face recognition and image recognition have achieved unprecedented success [David, Aja,Chris, 2016]. For example, in AlphaGo, a 19 × 19 checkerboard position is passed as an image, and a convolutional layer is used to construct a representation of the position for machine learning [Lawrence, Giles, Tsoi, 1997] [Yann, Yoshua, Geoffrey, 2015] .

Based on the basic concepts of symbolic logic and set theory, this paper focuses on judgment and attempts to provide a new method for the study of logic. It is hoped that graphic interpretation becomes a new research object of image recognition and machine learning. The rest of the paper runs in outline as follows: Section 2 briefly introduces predicates and set theory employed in this article; Predicate description of the main concept of judgment and the establishment of the judgment expansion J *are presented in Section 3; Section 4 describes the set theory description of J *; The graphic deduction of J * is carried out in section 5; Section 6 goes into a conclusion.

2 A Brief Introduction to Predicates and Set Theory

2.1 Predicates and quantifiers

Predicates are used to characterize nature and relation of individuals. For example, $P(x)$ means 'x has a property P', where P is the symbol which stands for a predicate and x an individual variable. For example, let P stand for 'animal' and a stand for 'cow' (a is an individual constant), then $P(a)$ means 'cow is an animal'.

Two kinds of quantifiers have been introduced into symbolic logic: The symbol ∀ translates 'all', and is called a universal quantifier; ∃ translates 'exists' and is called an existential quantifier. Then ∀x means 'all x' and $\exists x$ means 'some x'.

Intuitively, the semantics of 'some people are not students' and 'not everyone is a student' in natural language are the same. This makes us feel that there must be some relation between universal quantifier and existing quantifier. Let predicate H represent 'human' and A represent 'animal', then the above two sentences can be translated into symbols as:

$$\text{a. } \exists x(H(x) \wedge \neg S(x))$$

$$\text{b. } \neg\forall x(H(x) \rightarrow S(x))$$

b can be transformed into:

$$\text{b'. } \neg\forall x\neg\ (H(x) \wedge \neg S(x))$$

Obviously, $\exists x$ in a is replaced by $\neg\forall x\neg$ in b. Therefore, "$\exists x$" and "$\neg\forall x\neg$" are equivalent.

2.2 A brief Introduction to Naive Set Theory

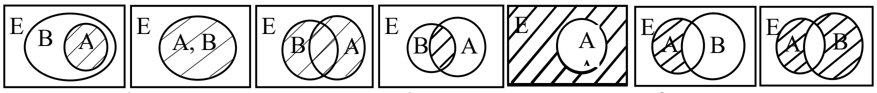
Set refers to collection of things with the same nature, which is represented by curly braces. Contents enclosed in curly braces separated by commas are called elements of the set. For example, if the set of positive integers is represented as **I**, then **I**= {1, 2, 3, … }.

There are two kinds of basic symbols in set theory. One is the relator between sets, and the other is the operator of sets. There are two basic relators. One is '∈', which means that an individual is an element of a set. The other is '⊆', meaning that all the elements of a set are elements of another set.

There are two basic operators. One is '∪', which means to combine the elements of the two sets together. The other is '∩', meaning that same elements in both sets are taken out and put together.

There is also a symbol ∅, called empty set, which means that there is no element in the set.

Figure 1 shows the Venn Diagram representation of basic relation and operation between sets. '*E*' represents universal set, which means all the sets in the domain. *~A* of Figure1e is read as the complementary set of A, the part that does not contain *A*. '~'is read as complementary operation: $\sim A=E-A$. *A-B* of Figure1f contains only the part of *A*, and does not contain the part of B: $A-B=A\cap\sim B$; '−' is read as difference operation, and '+' of Figure 1g is read as symmetric difference.



a. $A\subset B$ b. $A\subseteq B\wedge B\subseteq A$ c. $A\cup B$ d. $A\cap B$ e. $\sim A$ f. $A-B$ g. $A+B$

Figure 1: The Venn graphic representation of sets

3 Judgement J and J*

3.1 Nature Judgement J

Judgment' here refers to non-modal judgments in formal logic, which include universal affirmative judgment (**A**), universal negative judgment (**E**), particular affirmative judgment (**I**), and particular negative judgment (**O**). In formal logic, the description of **A, E, I** and **O** are as follows [Yuelin, 2006]:

The form of universal affirmative judgment is 'All S are P' (represented by **A**). The form of universal negative judgment is 'All S is not P' (represented by **E**). The form of particular affirmative judgment is 'S is P' (represented by **I**). The form of particular negative judgment is 'S is not P' (represented by **O**).
Translating **A**、**E**、**I** and **O** into symbols, we get**:**

A: $\forall x(S(x)\rightarrow P(x))$

E: $\forall x(S(x)\rightarrow\neg P(x))$

I: $\exists x(S(x)\wedge P(x))$

O: $\exists x(S(x)\wedge\neg P(x))$

Mathematically, **A**, **E**, **I**, and **O** are binary functions, and their range is $\{T, F\}$. For example, the form of universal affirmative judgment can be expressed as **A**($S(x)$, $P(x)$). Considering the habit of order and logical operator representation, we will represent **A**($S(x)$, $P(x)$) for S**A**P, and similarly: S**E**P, S**I**P and S**O**P.

There are two other nature judgments concerning individuals, which are described as follows:

The judgment of a singular affirmative judgment is the judgment of a certain individual thing having a certain character. The judgment of a singular negation judgment is the judgment of a certain individual thing not having a certain character [Yuelin, 2006].

Let '*a*' be a thing, where '*a*' is an individual constant, then $S(a)$ means '*a* has a property of *S*'. This is the symbolic description of a singular positive judgment. Similarly, the symbolic description of a singular negative judgment is represented as $\neg S(a)$.

We put together the above symbolic description of judgments, and make it J.

3.2 J* and Its Argument

A, E, I, O in **J** have a clear definition, which limits the scope of its research and application. To meet the extension of **J**, we need to design a new symbol. We let the four new symbols having the same semantics with **A, E, I** and **O** be **a, e, i, o**, and call them judgment words and make **J*** an extension of **J**.

The formal language of **J*** consists of two parts.

(1) symbols

a. constants: a, b; $a_1, a_2, \ldots, a_n$

b. variables: $x, y, z; x_1, x_2, \ldots, x_n$

c. predicates: $P, Q, S; P_1, P_2, \ldots, P_m$

d. logical connectives: $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$

e. judgment words: ***a***, ***e***, ***i***, ***o***

g. technical symbols:), (.

(2) Generate rules of well-formed formula

a. If $p\in\{P, Q, S, P_1, P_2, \ldots, P_m\}$, $c\in\{a, b; a_1, a_2, \ldots, a_n\}$, $v\in\{x, y, z; x_1, x_2, \ldots, x_n\}$, $\alpha\in\{p(c), p(v)\}$, then α is a well-formed formula.

b. if α is a well-formed formula, then $\neg\alpha$ is a well-formed formula.

c. if α, β are well-formed formulas, then $\alpha\wedge\beta$, $\alpha\vee\beta$, $\alpha\rightarrow\beta$, $\alpha\leftrightarrow\beta$ are well-formed formulas.

d. if α, β are well-formed formulas, then $\alpha\,\boldsymbol{a}\,\beta$, $\alpha\,\boldsymbol{e}\,\beta$, $\alpha\,\boldsymbol{i}\,\beta$, $\alpha\,\boldsymbol{o}\,\beta$ are well-formed formulas.

e. All well-formed formulas are merely *a*, *b*, *c* and *d*.

Built on this, the number of statement forms with judgment words can be expanded from 4 kinds of **J** to 32 kinds of **J***, which are then constrained to 8 kinds. We call these statement forms judgment patterns. To further extend the modes of the eight kinds of statement forms, we get:

$$\alpha\,\boldsymbol{a}\,\beta: \forall x(\alpha(x)\rightarrow\beta(x)) \quad (1)$$

$$\alpha\,\boldsymbol{e}\,\beta: \forall x(\alpha(x)\rightarrow\neg\beta(x)) \quad (2)$$

$$\alpha\,\boldsymbol{i}\,\beta: \exists x(\alpha(x)\wedge\beta(x)) \quad (3)$$

$$\alpha\,\boldsymbol{o}\,\beta: \exists x(\alpha(x)\wedge\neg\beta(x)) \quad (4)$$

$$\neg\alpha\,\boldsymbol{a}\,\beta: \forall x(\neg\alpha(x)\rightarrow\beta(x)) \quad (5)$$

$$\neg\alpha\,\boldsymbol{e}\,\beta: \forall x(\neg\alpha(x)\rightarrow\neg\beta(x)) \quad (6)$$

$$\neg\alpha\,\boldsymbol{i}\,\beta: \exists x(\neg\alpha(x)\wedge\beta(x)) \quad (7)$$

$$\neg\alpha\,\boldsymbol{o}\,\beta: \exists \mathrm{x}(\neg\alpha(x)\wedge\neg\beta(x)) \quad (8)$$

3.3 The Argument Form of J*

The finite sequence of statements is called the argument form. The final form of the sequence is called the conclusion, and other forms of the statement are called the premises. In order to get the correct conclusion, the premise in the argument must be true, related, and compatible.

It is now possible to study the relationship between judgments. First, observe formulas (1) and (2). If the negative connective '¬' is added before β of formula (1), then formula (1) becomes:

$$\alpha\, \boldsymbol{a} \neg\beta: \qquad \forall x(\alpha(x) \rightarrow \neg\beta(x))$$

Therefore, we get the equation $\alpha\, \boldsymbol{a} \neg\beta = \alpha\, \boldsymbol{e}\, \beta$. If '¬' is added before β of equation (2), we get the equation $\alpha\, \boldsymbol{e} \neg\beta = \alpha\, \boldsymbol{a}\, \beta$. A similar observation of formulas (4), (5) and (6), (7), (8), we get a bunch of equations:

Formulas Theorme **(FT).**

FT 1. $\alpha\, \boldsymbol{a}\, \beta = \alpha\, \boldsymbol{e} \neg\beta$
FT 2. $\alpha\, \boldsymbol{e}\, \beta = \alpha\, \boldsymbol{a} \neg\beta$
FT 3. $\alpha\, \boldsymbol{i}\, \beta = \alpha\, \boldsymbol{o} \neg\beta$
FT 4. $\alpha\, \boldsymbol{o}\, \beta = \alpha\, \boldsymbol{i} \neg\beta$
FT 5. $\neg\alpha\, \boldsymbol{a}\, \beta = \neg\alpha\, \boldsymbol{e} \neg\beta$
FT 6. $\neg\alpha\, \boldsymbol{e}\, \beta = \neg\alpha\, \boldsymbol{a} \neg\beta$
FT 7. $\neg\alpha\, \boldsymbol{i}\, \beta = \neg\alpha\, \boldsymbol{o} \neg\beta$
FT 8. $\neg\alpha\, \boldsymbol{o}\, \beta = \neg\alpha\, \boldsymbol{i} \neg\beta$

Then, analyze the compatibility relationship between the formulas. For the sake of simplicity and ease of comparison, formulas (3), (4), (7), and (8) are equivalently converted into formulas (3a), (4a), (7a), and (8a) which are expressed by the universal quantifier.

$$\neg\forall x(\alpha(x) \rightarrow \neg\beta(x)) \tag{3a}$$
$$\neg\forall x(\alpha(x) \rightarrow \beta(x)) \tag{4a}$$
$$\neg\forall x(\neg\alpha(x) \rightarrow \neg\beta(x)) \tag{7a}$$
$$\neg\forall x(\neg\alpha(x) \rightarrow \beta(x)) \tag{8a}$$

According to the law of excluded middle, we get the following theorem:

Judgments Incompatible Theorem. If two judgments have the same antecedent and consequent, they are incompatible, if and only if the two judgments are ***a*** and ***o***, or ***e*** and ***i***.

This theorem is basic. According to the Judgments Incompatible Theorem, there are a number of logical inference rules:

Equivalence Theorem (ET).

ET 1. $\alpha\, \boldsymbol{a}\, \beta \Leftrightarrow \neg(\alpha\, \boldsymbol{o}\, \beta)$
ET 2. $\neg(\alpha\, \boldsymbol{a}\, \beta) \Leftrightarrow \alpha\, \boldsymbol{o}\, \beta$
ET 3. $\alpha\, \boldsymbol{e}\, \beta \Leftrightarrow \neg(\alpha\, \boldsymbol{i}\, \beta)$
ET 4. $\neg(\alpha\, \boldsymbol{e}\, \beta) \Leftrightarrow \alpha\, \boldsymbol{i}\, \beta$
ET 5. $\neg\alpha\, \boldsymbol{a}\, \beta \Leftrightarrow \neg(\neg\alpha\, \boldsymbol{o}\, \beta)$
ET 6. $\neg(\neg\alpha\, \boldsymbol{a}\, \beta) \Leftrightarrow \neg\alpha\, \boldsymbol{o}\, \beta$
ET 7. $\neg\alpha\, \boldsymbol{e}\, \beta \Leftrightarrow \neg(\neg\alpha\, \boldsymbol{i}\, \beta)$

ET 8. $\neg(\neg\alpha\, \boldsymbol{e}\, \beta) \Leftrightarrow \neg\alpha\, \boldsymbol{i}\, \beta$

The symbol '$\Leftrightarrow$' means 'equivalence', which belongs to natural language.

4 Set Representation of J*

As predicates can construct sets, set theory can be adopted to study the above eight kinds of statement forms.

If set $A=\{x|\alpha(x)\}$, $B=\{x|\beta(x)\}$, then (1), (2), (5), and (6) can be easily rewritten to be equivalent set expressions. However, it is not that easy to rewrite the rest forms. For example, if we rewrite (3) as $\exists x\ (x\in A \wedge x\in B)$, then it can be rephrased either as 'Some x belong to both A and B'or as: 'There are some x that belong neither to nor belong to B.' Let us take A for analysis. Set A consists of all x with property α. In other words, an element either belongs to A or does not belong to A, and there is no saying as part of x belonging (not belonging to) A. Although this is a misreading, it is necessary to avoid it, and we have the following resolution:

Subset A' (composed by part of elements of set A) is contained in A. For example, let $A=\{1, 2, 3, 4, 5\}$, $A'=\{2, 3\}$. Obviously, $A'\subseteq A$. Also, let $y\in A'$, then $y\in A$. Likewise, we can rewrite '$\exists x(\alpha(x)\wedge \beta(x))$' into '$\exists y(y\in A \wedge y\in B)$'. Here we get a reasonable and clear expression to avoid misreading.

Now, let us rewrite logic statement forms (1) to (8) into equivalent set expressions:

$$\forall x(x\in A\rightarrow x\in B) \tag{1'}$$
$$\forall x(x\in \mathrm{A} \rightarrow x\notin B) \tag{2'}$$
$$\exists y(y\in A \wedge y\in B) \tag{3'}$$
$$\exists y(y\in A \wedge y\notin B) \tag{4'}$$
$$\forall x(x\notin A \rightarrow x\in B) \tag{5'}$$
$$\forall x(x\notin A \rightarrow x\notin B) \tag{6'}$$
$$\exists y(y\notin A \wedge y\in B) \tag{7'}$$
$$\exists y(y\notin A \wedge y\notin B) \tag{8'}$$

Here $A=\{x|\alpha(x)\}$, $B=\{x|\ (x)\}$; $y\in A'\subseteq A$, or $y\in B'\subseteq B$.

Figure 2 shows Venn diagrams of Formulas (1') to (8'). In Venn diagrams with existential quantifier statements, circles representing two sets are crossed to show the feature of 'part'. Slashes represent the state of element x in A and B, and the symbol '$\times$' represents the state of element y in A and B.

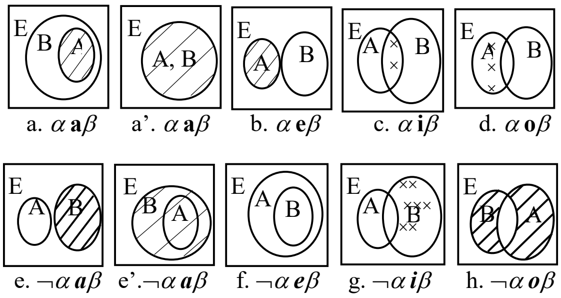


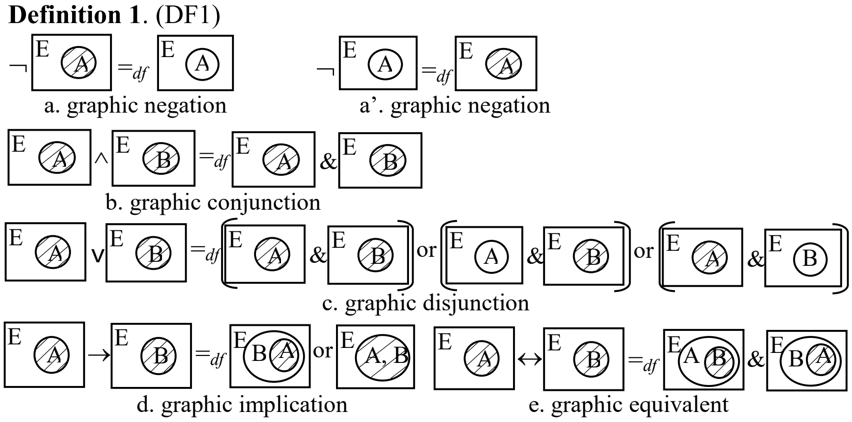
Figure 2: Venn graphic representation of ***a**, **e**, **i*** *and* ***o*** *modes.*

Figure 2a' shows A=B, which is a special case of Figure 2a. Although the two sets in Figure 2f are empty sets, they clearly indicate a inclusion relation.

5 The Graphic Argumentation of J*

To effectively demonstrate its graphic argument, we need to create graphic statement forms that fit the well-formed formula of **J***.

Definition 1. (DF1)

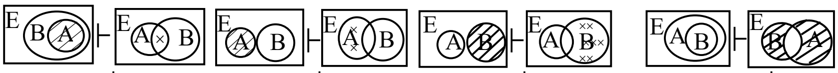


Here symbol "&" is the separator between premises.

Let us first observed Fig. 2a and Fig. 2c, which are both affirmative judgments. The slashes in Figure 2a occupy the entire A, while the slashes in Figure 2c occupy only part of A. This shows that: ***i*** is less certain than ***a***. See Figure 2b and Figure 2d, which are both negative judgments. We can get: ***o*** is less negative than ***e***. A similar analysis of Figure 2e and Figure 2g, Figure 2f and Figure 2h also leads to a corresponding conclusion. Due to the fact that the affirmative (negative) degree of universal judgments is weaker than particular judgments, we have the following conclusions:

Weakening theorem. If the premise and the conclusion of a judgment have the same antecedent and consequent, then ***a*** judgment pattern directly introduces ***i*** judgment pattern and ***e*** judgment pattern directly introduce ***o*** judgment pattern. According to weakening theorem, GT1 (Graphic Theorem) deduced from the graph can be obtained, where the symbol '$\vdash$' means 'introduce'.

GT1.



a. $\alpha\, \boldsymbol{a} \beta \vdash \alpha\, \boldsymbol{i}\, \beta$ b. $\alpha\, \boldsymbol{e}\, \beta \vdash \alpha\, \boldsymbol{o}\, \beta$ c. $\neg\alpha\, \boldsymbol{a} \beta \vdash \neg\alpha\, \boldsymbol{i}\, \beta$ d. $\neg\alpha\, \boldsymbol{a}\, \neg\beta \vdash \neg\alpha\, \boldsymbol{o}\, \neg\beta$

GT1d has a special premise, as it consists of two empty sets. Therefore, as for the degree of 'empty', the degree of conclusion is weaker than that of the premise.

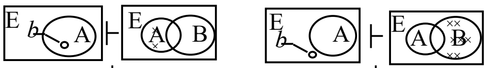
For clarity, let us use ST to represent a theorem corresponding to the symbolic description of GT. For example, ST1 corresponds to GT1.

As it is an individual deduction, existential quantifiers are easily thought of to be involved. For example, $\alpha(b)$ means that individual b has the property α. Since individual b is the instantiation of variable x, $\exists x \alpha(x)$ is true.

Strengthening theorem. If the individual is substituted into the judgment antecedent (consequent), so that it is true, then the corresponding ***i***, ***o*** is true.

According to strengthening theorem, there are:

GT2.



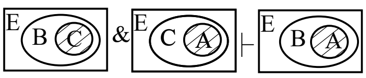
a. $\alpha(\boldsymbol{b}) \vdash \alpha\, \boldsymbol{i}\, \beta$ b. $\neg\alpha(\boldsymbol{b}) \vdash \neg\alpha\, \boldsymbol{i}\, \beta$

The argument discussed earlier has only one premise. In the argument of multiple premises, the premise must be compatible, and the premise must be related. If the premise is false and incompatible, the correct conclusion cannot be drawn. If there is no connection between the premises, the argument cannot be deducted. Therefore, the connection between the premises is a necessary condition for argumentation.

Irrelevant Theorem. Let the number of predicates is i, and the number of judgement item be j. The premises are irrelevant if and only if $j \geq i + 2$.

GT3. (**ST3**. $\delta\, \boldsymbol{a}\, \beta$, $\alpha\, \boldsymbol{a} \delta \vdash \alpha\, \boldsymbol{a}\, \beta$)

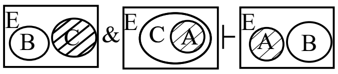
Since there is one more predicate, we need to construct the set: $C=\{x|\delta(x)\}$. According to the irrelevance theorem, there must be a connection between the premises, so it can be argued. First, draw the Venn diagram according to Figure 2.



Analysis: There is $A \subseteq C \wedge C \subseteq B$, so $A \subseteq B$.

GT4. (**ST4.** $\delta\, \boldsymbol{e}\, \beta$, $\alpha\, \boldsymbol{a} \delta \vdash \alpha\, \boldsymbol{e}\, \beta$

We first give a graphic theorem and then an analysis.



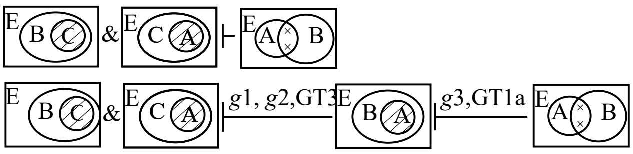
Analysis: There is $A \subseteq C \wedge C \cap B = \varnothing$, so $A \cap B = \varnothing \wedge A \cup B = A$.

GT3 and GT4 are basic forms of argument of Aristotle syllogism, so the use of graphical deduction can deduce others out.

We can use theorems we have obtained to deduce and arrive at new theorems.

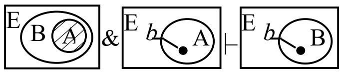
GT5. (ST5. $\delta\, \boldsymbol{a} \beta$, $\alpha\, \boldsymbol{a} \delta \vdash \alpha\, \boldsymbol{i}\, \beta$)

Proof:



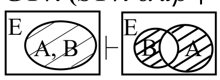
The symbol '$\vdash$' in the proof provides the basis for the next graph, where '*g*' means 'the premise graphic in the front of the symbol'$\vdash$''. The numbers that follow are serial numbers of graphs from left to right.

GT6. (ST6. $\alpha\, \boldsymbol{a} \beta$, $\alpha(b) \vdash \beta(b)$)



This is the famous syllogism, except that GT6's argumentation is simpler than the normative syllogism.

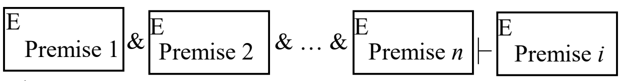
GT7. (ST7. $\alpha\, \boldsymbol{a} \beta \vdash \neg \alpha\, \boldsymbol{o} \beta$)



6 The Graphic Argumentation of J*

Based on the well-formed formula of the **J*** form language, let us extend the range of graphic deduction.

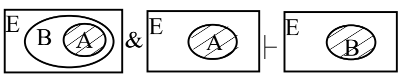
GT8.



where $1 \le i \le n$.

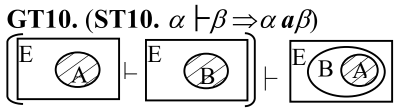
The correctness of GT8 is obvious, given that the premise in the argument form must be true and deduction fidelity must be guaranteed.

GT9. (**ST9.** $\alpha\, \boldsymbol{a} \beta$, $\alpha \vdash \beta$)

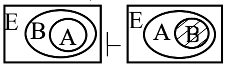


Analysis: There is $A \subseteq B \wedge A \neq \varnothing$, so $B \neq \varnothing$.

GT10. (**ST10.** $\alpha \vdash \beta \Rightarrow \alpha\, \boldsymbol{a} \beta$)

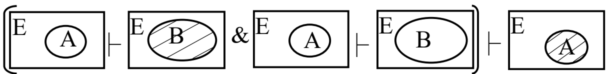


GT11. (ST11. $\neg\alpha\ \boldsymbol{a} \neg\beta\ \vdash \beta\ \boldsymbol{a}\ \alpha$)



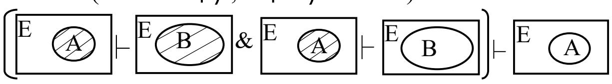
GT12. (**ST12.** $\neg\alpha \vdash \beta, \neg\alpha \vdash \neg\beta \Rightarrow \alpha$)

Proof:

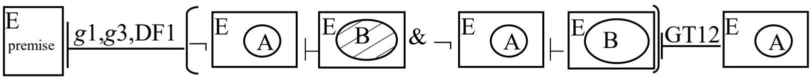


GT12 is called proof by contradiction. When the negative premise introduces two contradictory sub-conclusions, the positive premise becomes the conclusion.

GT13. (**ST13.** $\alpha \vdash \beta, \alpha \vdash \neg\beta \Rightarrow \neg\alpha$)

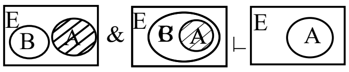


Proof:

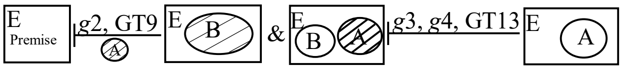


GT13 is called reductio ad absurdum; When an affirmative premise introduces two mutual-contradictory substatements, the negative premise becomes a conclusion. GT12 and GT13 are often used in proofs.

GT14. (**ST14.** α ***a***β, α ***e***β ⊢¬α)

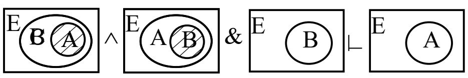


Proof:

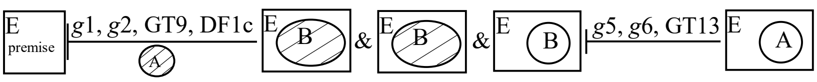


A hypothesis (assuming A is true) is added to GT14's proof. The conclusion is drawn using the method of reductio ad absurdum.

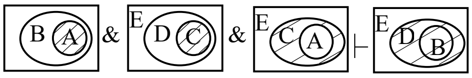
GT15. (**ST15.** $\alpha \leftrightarrow \beta, \neg\beta \vdash \neg\alpha$)



Proof：

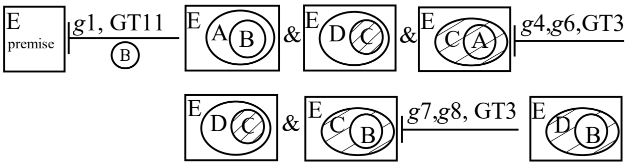


GT16. (**ST16.** $\alpha\ \boldsymbol{a}\beta, \delta\ \boldsymbol{a}\gamma, \neg\alpha\ \boldsymbol{a}\ \delta\ \vdash \neg\beta\ \boldsymbol{a}\ \gamma$)

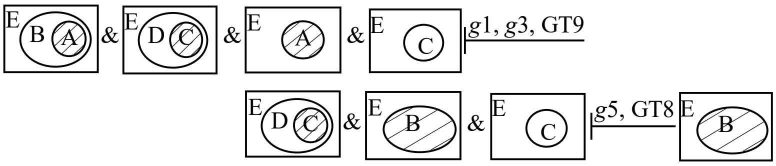


Where $D=\{x|\gamma(x)\}$.

Proof:



The equivalent form to ST16 is: $\alpha\, \boldsymbol{a}\beta$, $\delta\, \boldsymbol{a}\gamma$, $\alpha\vee\delta \vdash \beta\vee\gamma$. This is a dilemma. If α, δ are true, the theorem is true. However, α, δ may not be both true. The use of graphic argument contains two situations: $\alpha=T\wedge\delta=F$, $\alpha=F\wedge\delta=T$. The following is a proof with $\alpha=I\wedge\delta=F$ as the premise.



Another case is similar, and so omitted.

7 Conclusion

Set theory is recognized as a basic theory, which is widely used in many disciplines. Applying set theory to the study of logic will receive succinct, natural, and reliable results. In artificial intelligence, the understanding of natural language and the representation of knowledge are its basic research contents, which are closely related to judgment and deduction in form logic. Graphic theorem proposed in this paper involves non-modal deduction of form logic. Therefore, graphic deduction will be beneficial to the research of AI.

Acknowledgements

The first author would like to thank Department of Foreign Languages of Huaiyin Institute of Technology where she serves as a teacher and Graduate school of Chinese Academy of Social Science where she studies for a doctoral degree.

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