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# Maximum Capacity Overlapping Channel Assignment Based on Max-Cut in 802.11 Wireless Mesh Networks

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**Abstract:** By exploiting multi-radio multi-channel technology, wireless mesh networks can effectively provide wireless broadband access to the Internet for mobile users. Due to the limited number of orthogonal channels, overlapping channel assignment is one of the main factors that greatly affect the network capacity. However, current results in this area are not so satisfying. In this paper, we first propose a model for measuring achieved network capacity in MR-WMNs. Then we prove that finding an optimal overlapping channel assignment in a given MR-WMN with odd number of channels, is equivalent to finding an optimal assignment by only using its orthogonal channels. This theory allows us to use fewer channels to solve complicated channel assignment problems. Third, we prove that in 802.11b/g MR-WMN the simplified optimization problem is a Max-3-Cut problem. Although this problem is NP-hard, it has an efficient approximation algorithm that achieves approximation ratio of 1.19616 probabilistically by using the algorithm for Max-Cut whose approximation ratio is 1.1383 probabilistically. Based on the algorithm for Max-Cut, this paper proposes Max-Cut based channel assignment (MCCA) which uses a heuristic method to adjust the result produced by the Max-Cut algorithm to achieve an even better result. Finally, we perform extensive simulations to compare the MCCA with a state-of-the-art Tabu-Search based algorithm. The results show that the Max-Cut based overlapping channel assignment algorithm effectively and efficiently improves on the network capacity compared with existing algorithms.

**Keywords:** multi-radio multi-channel wireless mesh network; overlapping channel assignment; capacity optimization; graph coloring; Max-Cut
**Categories:** C.2.1, C.2.5, C.2.6, G.1.6, G.2.3

# 1 Introduction

Wireless mesh networks (WMNs) have been deployed in many areas for accessing the Internet wirelessly. They can support a variety of applications such as non-line-of-sight communication, indispensible resource sharing among large scale WLANs, and high-speed VoD, etc. Moreover, the wireless broadcasting nature facilitates the network deployment and connectivity to some extent. The primary components of a typical WMN include stationary wireless mesh routers and mobile wireless clients. Due to the self-organizing nature of mesh routers and the self-maintaining nature of network backbone [Akyildiz, 05], WMN can support mesh clients with reliable multi-hop connectivity in wide areas. Routers which have capabilities of accessing Internet are the gateways and they can transmit data between wireless and wired networks. A typical 802.11 WMN is illustrated in Figure 1. While WMN is being used more and more in metropolitan areas, a proven and noticeable weakness of WMN is that its aggregated throughput of a single-channel multi-hop network is limited [Gupta, 00]. In order to improve the network performance, researchers try to exploit multi-radio multi-channel technology which equips each mesh router with multiple radio interfaces, i.e. multiple network interface cards (NICs) that can be tuned to multiple channels. By allowing concurrent communication among multiple channels, the multi-radio wireless mesh network (MR-WMN) can greatly improve on the network capacity and broadband Internet accessibility over its counterpart, namely the single channel WMN.

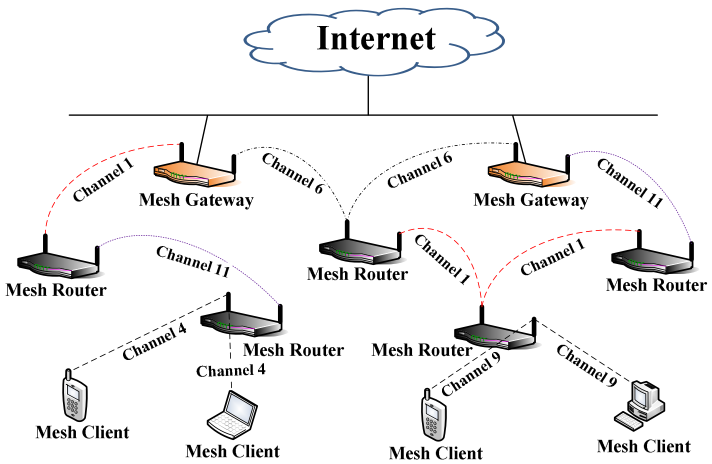



*Figure 1: A typical 802.11 multi-radio multi-channel WMN with seven routers, two radio NICs per router, and five frequency channels*

Although multi-radio technique can potentially improve the network performance, a primary challenging problem in deploying MR-WMN is how to efficiently assign the limited available channels to each mobile unit so that the network capacity can be maximized [Liu, 08]. For example, according to the FCC Rules and Regulation, the available frequency bandwidth for IEEE 802.11b/g network is quite limited and sequentially divided into 11 channels (frequency bands) numbered from 1 to 11. Two channels are overlapping or non-orthogonal if their channel numbers are close. According to [Hoang, 08] [Zeng, 10], channels need be separated by four or more channels to be considered orthogonal (non-overlapping). Two communicating pairs (links) can communicate concurrently without interfering each other if the two pairs use two orthogonal channels; otherwise, they would interfere each other [Cheng, 08] if any receiver is within the other pair's transmission range, which occurs frequently in an area shared by many mobile units. More close channel numbers are, more severe the interference will be. Such channel conflicts can cause link access collision, packet loss and result in serious link capacity reduction, which would degrade the quality of services at upper layers and lead to unpleasant user experience. Furthermore, due to the limited number of orthogonal channels, overlapping channels have to be assigned in large-scale MR-WMNs. Therefore, a key issue in deploying an MR-WMN is how we can assign a channel number to each link such that links within the others' transmission range are given orthogonal channel numbers. In case of impossible to achieve perfect orthogonality, we would like to have an overlapping channel assignment such that the total interference level is minimized, or equivalently, the total network capacity is maximized.

In this paper, we study the network capacity optimization problem and propose a low-interference overlapping channel assignment scheme for MR-WMNs. First, we present a channel assignment model that characterizes and measures, for a given channel assignment, the total amount of potential interference among overlapping channels in the network. Second, we show that, any optimal channel assignment MR-WMN with odd overlapping channels can be transformed to an optimal channel assignment with just orthogonal channels. Therefore, finding an optimal overlapping channel assignment in 802.11b/g MR-WMN is equivalent to finding an optimal assignment with 3 channels. This theoretical result simplifies the optimization problem in our study. Third, we further show that the 3-channel-assignment problem is NP-hard by showing that it is equivalent to the Max-3-Cut problem. This means that not only can these two optimization problems polynomially reduce to each other, but also an optimal solution for one problem directly maps to an optimal solution of the other. Since the Max-3-Cut problem has been well studied and has a highly efficient approximation algorithm that uses the well-known Max-Cut algorithm which achieves approximation ratio of 1.1383 probabilistically. Based on this algorithm, this paper proposes a heuristic method that adjusts the result produced by this algorithm to achieve an even better result. We give the proposed algorithm a name Max-Cut-based Channel Assignment (MCCA) algorithm. Finally, we present the results from extensive simulations that compared the MCCA with existing algorithms. The results show that the MCCA algorithm effectively improves on the network capacity over a well-known existing algorithm.

The rest of the paper is organized as follows. In Section II, we introduce related work. In Section III, we describe the system model adopted for measuring network capacity and define our optimization problem. In Section IV, we prove equivalence of the optimization problem and the Max-3-Cut problem. The channel assignment algorithm is presented in Section V. Section VI presents and analyzes simulation results, and Section VII concludes this paper.

## 2 Related Work

Channel assignment is one of the most attractive research topics in the field of multi-radio multi-channel wireless mesh networks. Sharma and Chaudhari [Sharma, 11] studied channel assignment problem with the objective of minimizing the channel interference. They regarded this optimization problem as Graph Colorability Problem (GCP) and then transformed it into a 3-CNF-Satisfiablility Problem. In addition, they illustrated it by one of the instances of graph coloring into 3-CNF-SAT. Marina *et al* [Marina, 10] proposed a topology control approach to efficiently achieve channel utilization. They formulated an integer linear programming (ILP) problem for obtaining a lower bound on the optimal channel utilization. In [Subramanian, 08], Subramanian *et al* addressed the problem of minimizing network interference among multi-radio nodes. They used a semi-definite programming approach to establish a lower bound and developed a heuristic polynomial algorithm to achieve good results. Raniwala et al [Raniwala, 05] proposed a distributed load-aware channel assignment algorithm for Hyacinth architecture, which was a logical gateway-root tree topology for multi-channel WMN. In the architecture, each node assigned the channels that were less used by its neighbouring nodes to its DOWN-NICs, and the channel assignment of the UP-NIC was the responsibility of its parent. However, all the above work assumed all channels are perfectly orthogonal which is not realistic for systems such as 802.11b, where most channel pairs are overlapping channels.

Hence, a number of researchers tried to solve the issue of partially overlapping channel assignment. Mishra et al [Mishra, 06] proposed a partially overlapping channel model for WMN, using partially overlapping channels to improve the utilization of the wireless spectrum. In [Rad, 06], Rad et al proposed a joint optimal channel assignment and congestion control (JOCAC) algorithm, which allocates partially overlapping channels to control the interference on each link, regarding the link's average congestion price. Liu et al [Liu, 10] studied load-aware assignment of partially overlapping channels, and presented a novel interference metric, which is defined to be the combination of overlapping degree between channels and channel utilization ratio of interfering nodes. In [Zhou, 12], Zhou et al address the partially channel assignment for 802.11b/g WLAN using SINR (Signal to Interference plus Noise Ratio) model instead of binary interference model. It considered the accumulative interference of the ambience of the receiver. However, due to this accumulative nature of interference, its combinatorial optimization technique is not applicable for protocol interference model. Cui et al [Cui, 11] studied partially overlapping channel assignment in 802.11 wireless networks. Based on the model of node orthogonality, they took into account both the adjacent channel separation and the physical distance of the two nodes employing adjacent channels. Its channel assignment algorithm, MICA, is based on a computational interference factor *Ic*. Their heuristic algorithm is effective, yet it is only suitable for WLAN whose traffics are downward. As for MR-WMN whose traffics are bidirectional, MICA is incapable of obtaining maximum orthogonality. Duarte et al [Duarte 12] proposed a distributed channel assignment algorithm by exploiting partially overlapped channels from a game theoretical approach, which modelled the interactions among mesh routers as a de-centralized game and derived a negotiation based optimal channel assignment based upon the properties of the potential game.

As discussed above, the existing channel assignment approaches are classified into two categories in general: centralized and distributed. In [Si, 10], Si *et al* pointed out that centralized approaches are capable of getting the optimal or near-optimal results, while distributed approaches are capable of quickly adapting to network changes and failures. Since mesh routers are stationary and traffic pattern is relatively static, in this paper we focus on centralized approaches to obtain better optimization results.

## 3 System Model and Problem Formulation

Our study focuses on 802.11 MR-WMN, because it is one of a currently prevalent form of WMNs [Capone, 10]. Since 802.11 technology has its origins in a 1985 ruling by FCC that released the ISM band for unlicensed use, we assume IEEE802.11 or 802.11b/g protocol using ISM band is employed at the physical layer to support upper layer applications. Further, we assume a WMN backbone is formed by stationary mesh routers and each router is equipped with a number of radio NICs working at 2.4GHz frequency band. For convenience, a router is also called a mesh node or node.

A wireless mesh network topology usually modelled by an undirected graph $G(V, E)$, where vertex set $V$ represents the set of mesh nodes and edge set $E$ represents the set of physical links. There is a bidirectional link between two nodes if they are within communication range. Let $C = \{1, 2 \ldots, k\}$ be the set of all available channels and $|C| = k$. The 2.4GHz frequency band is sequentially divided into 11 overlapping channels in North America or 13 channels in Europe, respectively [Mesh, 11]. Thus, $|C| = 11$ and $|C| = 13$ respectively for these two regions. Each node $u$ is equipped with multiple radio interfaces, one for each link $(u, v)$ associated with $u$. Each link $(u, v)$ is assigned a specific wireless channel from set $C$ to support communications between $u$ and $v$.

According to the previous works [Hoang, 08] [Subramanian, 07] [Zeng, 07] [Zeng, 10], when two links are within interference range, the amount of interference between two links depends on their channel separation, which is defined to be the difference of their channel numbers assigned. Channels separated by four or more are considered orthogonal (non-overlapping) and can communicate concurrently without interfering with each other. Zeng *et al* [Zeng, 10] further showed that the interference factor is linearly but inversely correlated to the channel separation. The precise relation [Zeng, 10] is showed in Figure 2.

In order to find an optimal channel assignment such that the total amount of interference is minimized, we need a model to measure the total interference for a given channel assignment. Let $G(V, E)$ be an 802.11b/g network. Let links in $E$ be labeled from $e_1$ to $e_m$, where $m = |E|$. Moreover, each link $e_i$, where $1 \leq i \leq m$, is assigned a channel $c_i$, $1 \leq c_i \leq 11$. We measure the interference level of this channel assignment as follows.

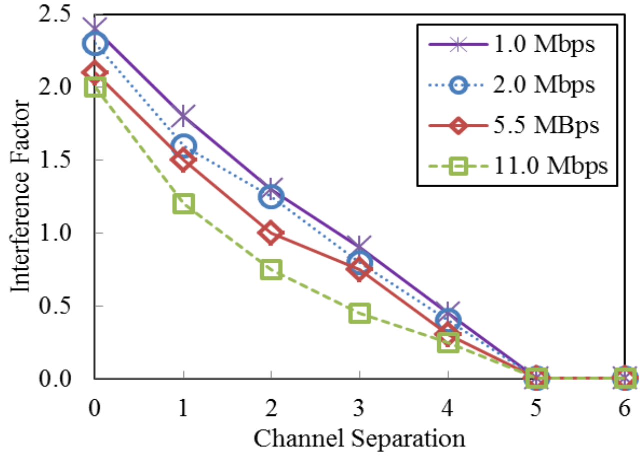



*Figure 2: Channel interference factor is inversely proportional to channel separation*

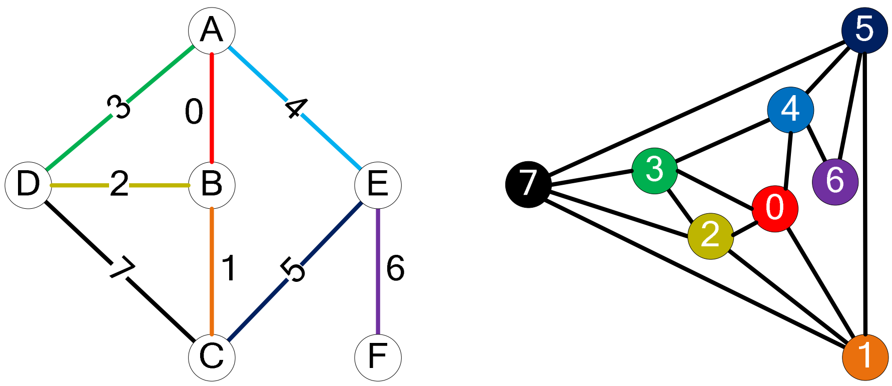
First, we construct a new graph $G'(V', E')$ called *interference graph*, where each node in $V'$ represents a link in $G$, $V' = \{v_i = e_i \mid e_i \in E\}$. An edge $(v_i, v_j)$ is included in set $E'$ if link $e_i$ and link $e_j$ are within interference range. Second, for a given channel assignment $A$, if link $e_i$ is assigned with channel $c$, we assign number $c$ to node $v_i$ in $G'$. Moreover, we assign a weight $w(v_i, v_j)$ to each edge $(v_i, v_j) \in E'$ in the interference graph to reflect the interference between links $e_i$ and $e_j$. Because the communication capacities of these two links are inversely affected by the interference which in turn is inversely proportional to the channel separation, we use the channel separation as the weight on the edge. If their separation is large than 5, we assign a weight 5, because two orthogonal channels reach their full capacity and will not go beyond this level even with a larger separation. Therefore, the weight is $w(v_i, v_j) = \min\{| c_i - c_j |, 5\}$. Let us call the value of $w(v_i, v_j)$ the *orthogonality* between links $e_i$ and $e_j$. Finally, the following formula is used to measure the total orthogonality of the network achieved by the channel assignment $A$:

$$T(G, A) = \sum_{(v_i, v_j) \in E'} w(v_i, v_j) \tag{1}$$

***Remark 1*** *Given a different channel assignment, the value of formula (1) may be different, but the interference graph G' remains the same.*

*In the interference graph,* $(v_i, v_j) \notin E'$ *means links* $e_i$ *and* $e_j$ *are far away, which is equivalent to being orthogonal and should be given a weight 5. Since the number of these missing edges in G' is a constant, the sum of weights on all these edges would be a constant, we omit these edges from the graph G' and omit their weight sum from formula (1). In other words, formula (1) reflects the total orthogonality for every pair of links in graph G.*

***Remark 2*** *We try not to use the term of total separation because a separation larger than 5 is not used. The orthogonality is a better wording.*




(a) An original network graph G (b) The interference Graph G'

*Figure 3: An example of interference graph*

Figure 3 shows an example of interference graph. Figure 3(a) is a mesh network *G* with 6 nodes and 8 links. Each node is equipped with multiple radio NICs, one for each link interface. Each link in Figure 3(a) is given a label from 0 to 7. The interference graph *G'* is shown in Figure 3(b), where 8 nodes correspond to the 8 links in network *G*. Each edge ($u$, $v$) in graph *G'* means that links $u$ and $v$ in original network *G* may interfere each other if they are assigned with non-orthogonal channels. Note that in construction of graph of 3(b), we assume two links in *G* would interfere each other if they are adjacent. This is just for illustration purpose. For real network, this should be determined by their actual physical locations. But, as can be seen later, our method will work for any definition of interference relation.

Now, based on above model, let us define the channel assignment problem.

***Definition 1*** *Given a network G and a set C of k available channels, the Capacity Optimization Problem (COP) is to find a channel assignment A such that the value of formula (1) is maximized in its interference graph G'. Such an assignment A is called a maximum capacity channel assignment.*

We call the COP problem a $k$-COP problem if $|C| = k$, which is a general problem for any WMN. For our 802.11 MR-WMNs, we need to make necessary assumptions on the set $C$ of available channels.

***Definition 2*** *A set C of k available channels is called normal if it satisfies the following conditions:*

1. *$|C| = k$ is an odd number;*
2. *The k channels are sequentially numbered from 1 to k;*
3. *Two channels are orthogonal if their channel separation is 5 or larger, which means that network with k available channels owns $(k + 4) / 5$ orthogonal channels, denoted as x.*

***Definition 3*** *Given a COP problem, if the set C of k available channels is normal, the COP problem is called a normal k-COP problem.*

In this paper, since we focus on 802.11 MR-WMN, for briefness we refer to normal *k*-COP as *k*-COP hereafter.

## 4 The equivalence Between *k*-COP Problem and The Max-*x*-Cut Problem

In this section we show that the *k*-COP problem is NP-hard. A key step is to show that a *k*-COP problem is equivalent to an *x*-COP problem where *x* denotes the number of orthogonal channels. The meaning of equivalence will be precisely explained. For ease of presentation, here we discuss 802.11b/g MR-WMNs where $k = 11$ and $x = 3$.

***Lemma 1*** *If A be a maximum capacity channel assignment for a given network G with a normal set C of 11 available channels, then, there exists a maximum capacity channel assignment A* for G that uses only 3 orthogonal channels 1, 6, 11.*

***Proof*** We will show how to transform the optimal assignment *A* to another optimal assignment *A** that uses only 3 orthogonal channels 1, 6, 11. Suppose assignment *A* uses channel numbers other than 1, 6, 11, for otherwise, we are done. Let $c(v)$ denote the channel number assigned by *A* to node *v* in the interference graph $G'(V', E')$. We divide the nodes in $V'$ into 11 groups, $S_i$, $1 \le i \le 11$, according to their channel numbers. Specifically, we define $S_i = \{v \mid v \in V' \text{ and } c(v) = i\}$. Some groups may be empty. Thus, every node in $S_i$ is assigned with the same channel number *i*.

Let us make a simple observation first. Suppose $c(v) \notin \{1, 6, 11\}$, say $c(v) = 3$. If we change $c(v)$ to 2, how would the weight $w(u, v)$ of edge $(u, v) \in E'$ change? Depending on the channel number $c(u)$, we have 3 cases:

1. If $c(u) < 3$ then $w(u, v)$ reduces by 1;
2. If $3 < c(u) \le 7$ then $w(u, v)$ increases by 1;
3. If $8 \le c(u)$ then $w(u, v)$ does not change.

Now, consider the set $S = S_3 \cup S_8$. If we reduce the channel number by one for every node in *S*, how much the value would change in the formula (1)?

Let $S_{\text{low}} = S_1 \cup S_2$, $S_{\text{mid}} = S_4 \cup S_5 \cup S_6 \cup S_7$, $S_{\text{high}} = S_9 \cup S_{10} \cup S_{11}$. Moreover, let

$$E'_1 = \{(u, v) \mid (u, v) \in E', u \in S_{\text{low}}, v \in S_3\};$$
$$E'_2 = \{(u, v) \mid (u, v) \in E', u \in S_3, v \in S_{\text{mid}}\};$$
$$E'_3 = \{(u, v) \mid (u, v) \in E', u \in S_{\text{mid}}, v \in S_8\};$$
$$E'_4 = \{(u, v) \mid (u, v) \in E', u \in S_8, v \in S_{\text{high}}\}.$$

It is easy to see that the new value of formula (1) would be

$$T(G, A) - |E'_1| + |E'_2| - |E'_3| + |E'_4|.$$

Because T(G, A) is maximized, we have

$$-|E'_1| + |E'_2| - |E'_3| + |E'_4| \le 0. \tag{2}$$

Now, instead of reducing channel numbers, if we increase the channel number by one for every node in *S*, how much the value would change in the formula (1)? It is easy to see that the new value of formula (1) would be

$$T(G, A) + |E'_1| - |E'_2| + |E'_3| - |E'_4|.$$

Because $T(G, A)$ is maximized, we have

$$|E'_1| - |E'_2| + |E'_3| - |E'_4| \leq 0. \quad (3)$$

From inequalities (2) and (3), we have

$$|E'_1| - |E'_2| + |E'_3| - |E'_4| = 0.$$

Therefore, no matter we increase or reduce the channel number by one for every node in $S$, the value of formula (1) remains maximized. In other words, this operation lead us to another assignment $A'$ which is also optimal. The difference is in the new assignment $A'$, the set $S_3$ and set $S_8$ will be empty.

From above observation, it is clear that we can transform the optimal assignment $A$ to another optimal assignment $A'$ such that set $S_3$ and $S_8$ will be empty. Then, we can transform $A'$ to another optimal assignment $A''$ such that $S_5$, $S_{10}$ are empty. Repeat this process two more times with $S_4$, $S_9$ and $S_2$, $S_7$ are empty, we can obtain an optimal assignment $A^*$ such that only $S_1$, $S_6$, and $S_{11}$ are non-empty. ▌

***Corollary 1*** *For the same 802.11b/g network G, finding an optimal solution of a k-COP and finding an optimal solution of 3-COP have the same time complexity.*
***Proof*** Given an optimal solution of a $k$-COP for a network $G(V, E)$, it is easy to see that the transformation used in the proof of Lemma 1 needs only $O(|V|)$ time because $k$ is a constant number. Moreover, any channel assignment needs at least $\Omega(|V|)$ time. Therefore, finding an optimal solution of a $k$-COP and finding an optimal solution of a $3$-COP for the same network $G$ have the same time complexity. ▌

Now, we discuss the relation between the 3-COP and Max-3-Cut problem. Let us state the Max-3-Cut problem first.

***Definition 4*** *Given a graph G(V, E), where vertex set V is partitioned into 3 sets, $V_1$, $V_2$ and $V_3$, the set P of edges between different vertex sets is called the 3-Cut for the partition, that is,* $P = \{(u, v) \mid (u, v) \in E, u \in V_i, v \in V_j, i \neq j, 1 \leq i, j \leq 3\}$.

***Definition 5*** *Given a graph G(V, E), the Max-3-Cut problem is to partition V into 3 sets, $V_1$, $V_2$ and $V_3$ such that the corresponding 3-Cut P has the largest size.*

The Max-3-Cut problem is a well-known NP-complete problem [Frieze, 97], but there is an efficient approximation algorithm for this problem [Frieze, 97] [Coja-Oghlan, 06]. Similarly, we can define a **Max-*x*-Cut** problem if the vertex set $V$ is partitioned into $x$ subsets. When $x = 2$, the Max-2-Cut problem is the well-known NP-hard problem known as the **Max Cut** problem.

***Theorem 1*** *The 3-COP problem is equivalent to the Max-3-Cut problem.*
***Proof*** On one hand, any channel assignment to the 3-COP problem induces a vertex partition of the interference graph $G'(V', E')$ as follows:

$$V_1 = \{v \mid v \in V' \text{ and } c(v) = 1\};$$
$$V_2 = \{v \mid v \in V' \text{ and } c(v) = 6\};$$
$$V_3 = \{v \mid v \in V' \text{ and } c(v) = 11\}.$$

This partition corresponds to a 3-Cut $P$.

Moreover, because edge $(u, v)$ in $E'$ has weight $w(u, v) = 0$, if $u$ and $v$ belong to the same set, and 5 otherwise, we have

$$T(G, A) = 5\,|P|, \text{ where } P \text{ is the corresponding 3-Cut.}$$

On the other hand, any 3-Cut $P$ of the interference graph $G'(V', E')$ induces a channel assignment $A$ for the interference graph $G'$ as follows. Let set $V'$ be partitioned into $V_1$, $V_2$ and $V_3$ by the 3-Cut. We assign every node in $V_1$ with channel 1; assign every node in $V_2$ with channel 6; and assign every node in $V_3$ with channel 11. Obviously, the value of formula (1) achieved by this assignment is

$$T(G, A) = 5\,|P|.$$

Now, we can see that the set of channel assignments for interference graph $G'$ and the set of 3-Cuts for the same graph $G'$ have one-to-one correspondence. Moreover, the size of a 3-Cut $P$ and the value $T(G, A)$ of its corresponding channel assignment are related by $T(G, A) = 5\,|P|$. Therefore, an optimal channel assignment corresponds to a Max-3-Cut, and vice versa. Therefore, a solution to one problem also directly produces a solution to the other problem, which means they are equivalent. ▌

***Corollary 2*** *Given an 802.11b/g network G, finding an optimal channel assignment for a k-COP problem and finding a Max-3-Cut for its interference graph G' have the same time complexity.*
***Proof*** This follows from Corollary 1 and Theorem 1. ▌

Obviously, the 3-COP problem is an NP-hard problem.

***Corollary 3*** *In any MR-WMN with k overlapping channels (k is odd) and x orthogonal channels, finding an optimal solution of a k-COP is equivalent to finding an optimal solution of x-COP.*
***Proof*** This corollary can be induced from Lemma 1. ▌

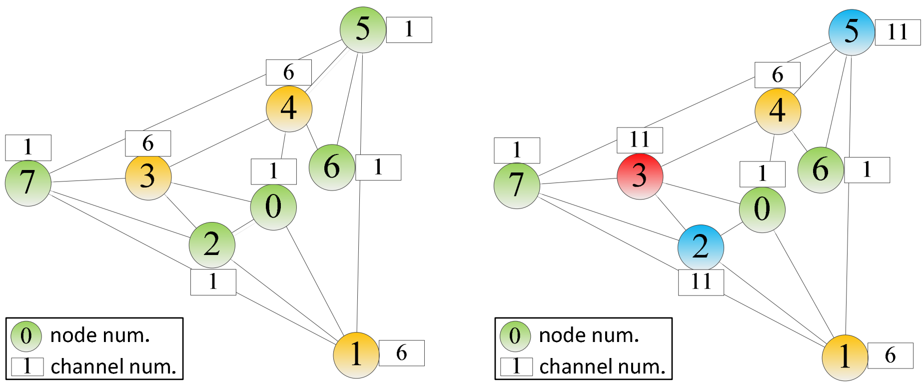
***Remark 3*** *Theorem 1 and Corollary 2, 3 do not mean that we only need x channels in 802.11 MR-WMNs. This theorem merely means that from network capacity point of view, we can achieve the maximum value of formula (1) by using just x channels. However, if we use only x channels, then two adjacent nodes with the same channel number will not be able to transmit concurrently at all, which is not desirable. An interesting approach to overcome this problem is that we can do opposite transformations discussed in Lemma 1. In other words, after we have achieved the maximum value of formula (1) by x channels, we can change the assignment by doing reverse transformations while keeping the value of formula (1) maximum to improve other network aspects such as connectivity, etc. This will be a part of our future work.*

## 5 Algorithm Design

Since 802.11b/g MR-WMN is one of a currently prevalent form of WMNs, this section focuses on how to efficiently find a near optimal solution to a given 3-COP problem for a 802.11b/g network $G$ which is equivalent to finding a near optimal solution to the Max-3-Cut problem for its interference graph $G'$. As we pointed out, there is an efficient approximation algorithm [Coja-Oghlan, 06] for the Max-3-Cut problem which has an approximation ratio of 1.1383 probabilistically. However, since this is a probabilistic ratio, it may not produce a guaranteed desirable result, especially when the size of the problem is not sufficiently large. We notice that the algorithm in [Coja-Oghlan, 06] relies upon a subroutine for Max-2-Cut problem, i.e. the well-known Max-Cut problem with probabilistic approximation ratio 1.19616. Following the same approach, our proposed algorithm *MCCA* also uses the Max-Cut algorithm as the base but uses a heuristic subroutine to adjust the result to achieve a better result. Detailed steps are explained below.

Suppose a solution to a 3-COP problem partitions the vertex set $V'$ of graph $G'(V', E')$ into 3 subsets, and assigns channels 1, 6, 11 respectively to the 3 sets. Now, if we view the three channels 1, 6, and 11 as three colors, then **this channel assignment can be viewed as assigning a distinct color** to each subset and all nodes in the same subset are colored with the same color. Now, the corresponding 3-Cut contains all edges whose endpoints are given different colors. Let us call other edges ***monochromatic*** edges because such an edge has the same color on its two endpoints. Thus, the Max-3-Cut is a 3-Cut that minimizes the number of monochromatic edges.

From the above node coloring point of view, our algorithm takes the following steps for a given network $G$, where the sub-routine named *ApproxMC* finds a near optimal *Max-Cut* using *SDP* (Semi-Definite Program) evaluation approach introduced in [Frieze, 97]. Our simulation implemented this algorithm. This algorithm has 1.1383 approximation ratio probabilistically.




(a) Colored graph after step 3 (b) The final channel assignment

*Figure 4: An example of the MCCA algorithm*

1. Construct the interference graph $G'(V', E')$ and invoke subroutine *ApproxMC*($G'$) to partition $V'$ into two subsets $L$ and $R$ which induce two subgraphs $G'_L$ and $G'_R$.
2. Color $L$ and $R$ with color 1 and color 6, respectively.
3. Invoke subroutine *ApproxMC* on graphs $G'_L$ and $G'_R$, respectively to partition set $L$ into subsets $LL$ and $LR$; and partition set $R$ into subsets $RL$ and $RR$. They induce graphs $G'_{LL}$, $G'_{LR}$, $G'_{RL}$, $G'_{RR}$, respectively.
4. Recolor nodes in $LR$ and $RR$ with color 11.
5. Compute number of edges between $LR$ and $RR$, which is denoted as $BN(11)$.
6. Compute number of edges between $LL$ and $LR$, which is denoted as $BN(1)$.
7. Compute number of edges between $RL$ and $RR$, which is denoted as $BN(6)$.

8. If $BN(1) < min\{BN(6), BN(11)\}$, then recolor $LR$ with color 1; else if $BN(6) < BN(11)$, then recolor $RR$ with color 6.
9. Check every vertex $v$, if recolor $v$ with another color can increase the orthogonality, then do so.

The pseudo codes for the subroutine as well as the main algorithm are presented below.

As an example, Figure 4 shows how the *MCCA* works on the graph of Figure 3(a). After step 3, nodes are partitioned into two subsets and colored with 1 and 6, which is shown in Figure 4(a), where $L = \{0, 2, 5, 6, 7\}$ and $R = \{1, 3, 4\}$.

**Algorithm 1 MCCA(Max-Cut-based Channel Assignment)**

**Input**: $G'$: interference graph, $C$: available channels
**Output**: $G'$: with nodes colored

```
ApproxMC(G');
assign color 1 to vertices in G'_L and color 6 to vertices in G'_R;
ApproxMC(G'_L);
ApproxMC(G'_R);
assign color 11 to vertices in G'_LR and G'_RR;
BN(1) ← the number of links between G'_LL and G'_LR;
BN(6) ← the number of links between G'_RL and G'_RR;
BN(11) ← the number of links between G'_LR and G'_RR;
if BN(1) < min{(BN(6), BN(11)}
   recolor vertices in G'_LR with color 1;
else if  BN(6) < BN(11)
   recolor vertices in G'_RR with color 6;
end if
V' ← the number of vertices in G';
for i = 1 to V'  // Adjust each node's color in global
   N(c) ← number of neighbors of v_i that has color c, c ∈{1, 6, 11};
   if N(c) = min{N(1), N(6), N(11)}
      color v_i with color c;
   end if
end for
return G';
```

**ApproxMC($G'$) // A sub function using Semi-Definite Program**

**Output**: $G'_L$,$G'_R$: Max-2-Cuts

```
CNT ← 9.9E299;  // CNT is a sufficiently large constant
V' ← the number of vertices in G';
E' ← the number of edges in G';
op1 ← V' × E' / (V' – 1) – 2 × V' × [2 × E' / (V' – 1)]^½;
op2 ← V' × E' / 2 / (V' – 1) + CNT × V' × [2 × E' / (V' – 1)]^½;
if E' ≥ op1 && SDP(G') ≤ op2
   for i = 1 to V'
      choose an vertex v which is not in G'_L and G'_R;
      N(1) ← count the number of v's neighbors which are in G'_L;
      N(6) ← count the number of v's neighbors which are in G'_R;
```

```
        if N(1) ≤ N(6)
            add vertex v to G'_L;
        else
            add vertex v to G'_R;
        end if
    end for
else
    for i = 1 to |V'|
        compute Max 2-Cut in S for every S ⊂ V', |S| = i, and
        partitions them into G'_L and G'_R;
    end for
end if
```

Next, the vertices of $G'_L$ and $G'_R$ are partitioned into 4 subsets, where $LL = \{0, 6, 7\}$, $LR = \{2, 5\}$, $RL = \{1, 4\}$, and $RR = \{3\}$. Since $BN(1) = 6$, $BN(6) = 3$ and $BN(11) = 1$, the algorithm colors {3, 2, 5} with 11. It is not difficult to verify that this channel assignment achieves the maximum orthogonality value which is 70. In this example, Step 9 makes no change because the result was optimal before Step 9.

The time complexity of *MCCA* can be analyzed as follows. Step 1 and step 3 take an expected polynomial time [Frieze, 97]. All other steps take $O(n + m)$ time, where $n$ and $m$ are the number of vertices and the number of edges in the interference graph $G'$. Therefore, *MCCA* has an expected polynomial running time. Our simulations measure the actual running time which shows that the *MCCA* runs quite fast even for up to 10000 nodes.

# 6 Evaluations

In this section, we provide simulation results and evaluate the performance of *MCCA* by comparing with a well-known and high efficient algorithm, the Tabu Search-based algorithm (Tabu-Search for short) [Subramanian, 08].

## 6.1 Overview

In order to evaluate the efficiency of the two algorithms, both algorithms are performed in a real environment on personal computers and their execution time in microseconds are measured. The runtime environment of algorithms is 64-bit Windows 7 Ultimate, and the processor and memory are Intel Core 2 Quad Q8200 @ 2.33GHz and DDR3 4.00GB, respectively. Moreover, the Java Runtime Environment (JRE) is version 6.00.update.26 of 64-bit.

Since Tabu-Search algorithm uses an iterative method to assign channels, it produces fluctuated results in different runs. Therefore, two algorithms are performed 100 times in each setting and the mean value is calculated for comparisons. In each run of the Tabu-Search, the number of round is fixed to fifty and the number of neighbouring solutions is fixed to one (See [Subramanian, 08]).

Before presenting our simulation result, let us introduce the technique of *backward engineering* for constructing the interference graphs in our simulation. In order to test how close to the optimum an algorithm can achieve, we need to compute the optimal results for comparison. However, because the problem is NP-hard, if we randomly generate a graph, it is difficult to obtain optimal results. Using the technique of backward engineering, we do the following. We start from 3 vertices, *a*, *b*, *c* and color them with 1, 6, 11, respectively. Then, we add one node *d* and connect it to *a* and *b*, and color it with 11. We can continue this way to add one more node in the graph. Each time, we randomly select a few nodes in current graph from two colors, connect them to a new node, and color the new node with the third color. By doing this way, we generate a random 3-partable graph whose Max-3-Cut is known. Then, we erase all colors from the graph and use the graph to test the two algorithms to see whether they can produce optimal assignment or not. If not, how close it is.

We use the above technique to generate first group of networks ranging from 3 to 50 nodes.

In addition to those network graphs generated by the backward engineering, we also use a special kind of graphs for testing, which is a set of complete graphs ranging from 3 nodes to 50 nodes. This is the second group of network graphs.

Third, we generate networks with large number of nodes ranging from 100 to 10000 by the backward engineering.

Moreover, we make comparisons in real network environment between *MCCA* and Tabu-Search by adopting the topology of MIT Roofnet for evaluation.

The above four types of networks are summarized in Table 1.

| **Senario No.** | **Description** |
|---|---|
| 1 | $1000 \times 1000m^2$ flat region, graphs with n ($3 \leq n \leq 50$) vertices and the optimal orthogonality is known in advance through backward engineering |
| 2 | $1000 \times 1000m^2$ flat region, complete graphs with n ($3 \leq n \leq 50$) vertices |
| 3 | $1000 \times 1000m^2$ flat region, large-scale interference graph with n ($100 \leq n \leq 10000$) vertices and the optimal orthogonality is known in advance |
| 4 | The real topology of MIT Roofnet with 70 nodes and 211 wireless links |

*Table 1: Network parameters used in tests*

### 6.2 Simulation Settings

In every setting, we test static 802.11b/g wireless mesh network with *n* nodes. Each radio NIC has a fixed transmission range of 250*m* and interference range of 450*m*. We set the theoretical link capacity of each link at the physical layer to 1.0 Mbps. Due to the relationship of orthogonality and network capacity, if two channels are orthogonal, the link capacity between them is 1.0 Mbps; if two channels are non-orthogonal, the link capacity between them is $|c_2 - c_1| / 5 \times 1.0$ Mbps. So we just use the total orthogonality to indicate the capacity for convenience. Both algorithms employ the orthogonal channel 1, 6, and 11 for channel assignment.

### 6.3 Simulation Results

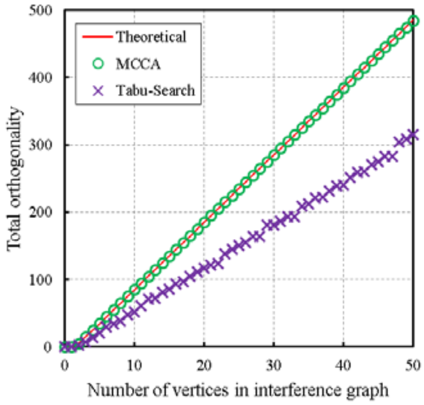



*Figure 5: Total orthogonality comparison in specific interference graph*

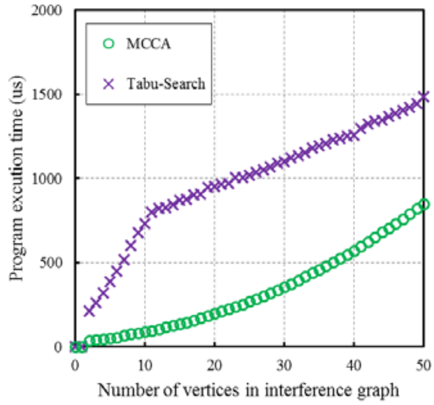



*Figure 6: Program execution time comparison in specific interference graph*

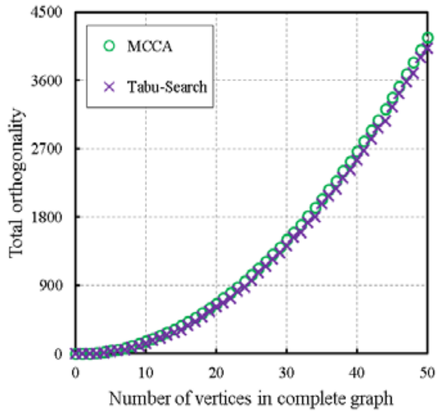



*Figure 7: Total orthogonality comparison in complete interference graph*

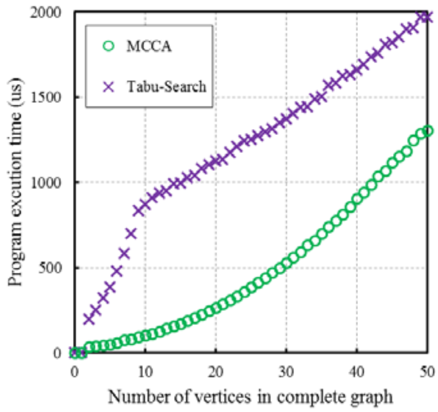



*Figure 8: Program execution time comparison in complete interference graph*

All simulation results are shown in Figures 5 – 12. Figure 5 illustrates the orthogonality comparison. As shown in Figure 5, with the increasing number of vertices, the network orthogonality achieved by both algorithms increase linearly. However, the growth rates are obviously different. The assignments produced by *MCCA* almost always achieve maximum orthogonality that matches with theoretical optimal value. By theoretical value we mean the maximum value in each case is known and calculated in advance. The Tabu-Search, however, cannot approach the theoretical value, and with increasing number of vertices, the performance gap becomes larger. For example, when the number of vertices is 50, the value derived by Tabu-Search is only 64.9% of the theoretical value.

We also compared the time complexity of these two algorithms by measuring actual time needed. Figure 6 shows the time needed by each algorithm in the testing for the first group. We can see that, *MCCA* is much faster than Tabu-Search. Though the execution time of both algorithms are gradually increasing, the growth rate of *MCCA* is smaller and the execution time is considerably small in all simulations. In addition to the execution time gap, the execution time curve of the *MCCA* is notably smoother, and this means the *MCCA* has better scalability as the wireless mesh network size expands.

Figure 7 shows the orthogonality comparison for the second group of graphs. As the network size increases, total orthogonality achieved by both algorithms increase dramatically from 5 to 4000, which is eight hundred times increase. This is due to the quadratically increased number of edges, which makes the orthogonality increase quadratically. As illustrated in this figure, the total orthogonalities derived by both algorithms are almost the same. This gives us significant information that because the complete graph provides a link between every two nodes and it is highly symmetrical so that either algorithm has no obvious advantages in finding the interference-free results.

Figure 8 shows the time needed by each algorithm in the testing for the second group. We can see that, as the network size increases, *MCCA* runs much faster than the Tabu-Search in all circumstances. In the extreme case, their difference is almost as large as 8.4 times. Even under the normal circumstances, the difference is still 1.5 times. This is because *MCCA* does not use iterative methods which need time to converge; instead, *MCCA* assigns channels more directly.

When jointly considering Figure 5 and Figure 7, we can observe that the curves in Figure 5 are growing with a smaller rate than that in Figure 7. This is because the growth rate of the number of edges in the network graphs of the first group is smaller than in that of the second group. From Figure 6 and Figure 8, we can discover that both algorithms use less time for first group of graphs than for the second group of graphs. It is clear from the tests of the scenario 1 and 2 that the number of edges in a graph is the major factor of computational time.

Figure 9 shows the orthogonality comparison for the third group of graphs. Note that the coordinate axes are both in log-arithmetic scale. It is interesting to see that, as the number of vertices increases, there is a perfect linear relation between the number of vertices and orthogonality. For example, as the number of vertices increases from 100 to 10000, the network capacity rises from 1000 to 100000. Both of them are equally increased a hundred times. This is because the number of links is increased at the same rate (on log-arithmetic scale). Each growth rate curve of the two algorithms has a linear relation to the theoretical value, but *MCCA* is much closer to the theoretical value in all cases, which indicates its effectiveness.

Figure 10 shows the time needed by these two algorithms in the testing for the third group of graphs. Note that the coordinate axes are both in log-arithmetic scale also. We can see that, as the number of vertices increases, both time needed by two algorithms are increased linearly. Interestingly, when the number of vertices is small, the time of Tabu-Search is somewhat smaller than that of the *MCCA*. However, when the number of vertices is large, as in most of the cases, the *MCCA* has an extraordinary advantage in execution time over the Tabu-Search. And as the number of vertices increases, the gap between two algorithms is growing larger and larger.

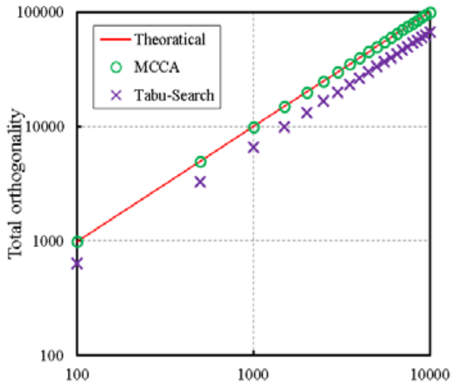



*Figure 9: Total orthogonality comparison in large-scale interference graphs*

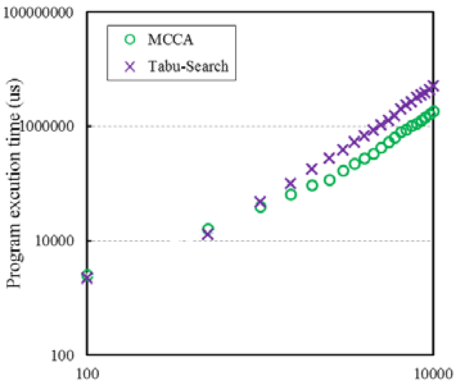



*Figure 10: Program execution time comparison in large-scale interference graphs*

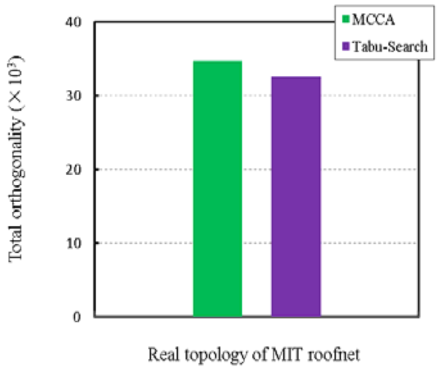



*Figure 11: Total orthogonality comparison under the topology of roofnet*

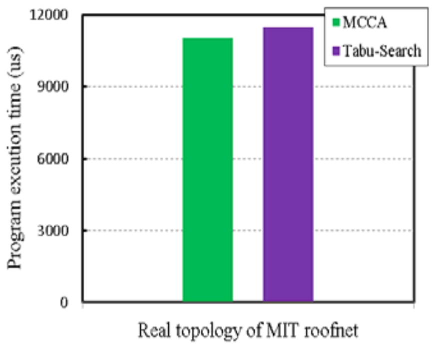



*Figure 12: Program execution time comparison under the topology of roofnet*

We perform the orthogonality and execution time comparisons using the MIT roofnet topology, which is derived from a real network [MIT, 12]. The results can be found in Figure 11 and Figure 12. As Figure 11 shows, the orthogonality gained by *MCCA* still surpasses the Tabu-Search, which proves the effectiveness of *MCCA*. Meanwhile, Figure 12 indicates that *MCCA* runs a little faster than Tabu-Search. This is because *MCCA* does not use iterative methods which need converge.

## 7 Conclusion

In this paper we have proposed a realistic capacity measuring model for MR-WMNs and formulated the *k*-COP channel assignment problem with the objective of maximizing network capacity. We prove that finding an optimal overlapping channel assignment in any MR-WMN with odd number of channels, is equivalent to finding an optimal assignment by only using its orthogonal channels. This theory considerably simplifies complicated overlapping channel assignment problems. Especially, the 3-COP problem is proved to be equivalent to the Max-3-Cut problem. Not only does this result proves the NP-hardness for the 3-COP problem, but also allows us to solve the channel assignment for 802.11b/g MR-WMNs by solving the Max-3-Cut problem.

Based on the highly effective well-known approximation algorithm for Max-Cut problem, we have proposed *MCCA* algorithm which uses a heuristic approach to further improve the channel assignment produced by the Max-Cut algorithm. Our extensive simulations show that *MCCA* algorithm which runs in polynomial expected time can produce a channel assignment for a given network that achieves optimal or near optimal network capacity efficiently.

In the future we will consider the distance impact between communication links. Such distance will cause transmission and interference power fading known as *Path Fading*. Such fading may lead to inaccurate results. In addition, we will study how to tune the obtained assignment by doing reverse transformations to improve other network aspects such as connectivity while keeping the network capacity unchanged.

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## References

[Akyildiz, 05] Akyildiz, I.F., Wang, X.D.: A survey on wireless mesh networks. IEEE Communications Magazine, 43 (9), 2005, S23- S30.

[Capone, 10] Capone, A., Carello, G., Filippini, I., Gualandi, S., Malucelli, F.: Routing, scheduling and channel assignment in Wireless Mesh Networks: Optimization models and algorithms. Ad Hoc Networks, 8 (6), 2010, 545-563.

[Cheng, 08] Cheng, H., Yang, S.X.: Joint Multicast Routing and Channel Assignment in Multiradio Multichannel Wireless Mesh Networks Using Simulated Annealing. Simulated Evolution and Learning, 2008, 370-380.

[Coja-Oghlan, 06] Coja-Oghlan, A., Moore, C., Sanwalani, V.: MAX k-CUT and approximating the chromatic number of random graphs. Random Structures & Algorithms, 28 (3), 2006, 289-322.

[Cui, 11] Cui, Y., Li, W., Chen, X.Z.: Partially overlapping channel assignment based on "node orthogonality" for 802.11 wireless networks. The 2011 IEEE International Conference on Computer Communications (INFOCOM'11), April 2011, 361-365.

[Duarte, 12] Duarte, P.B.F., Fadlullah, Z.M., Vasilakos, A.V., Kato, N. On the partially overlapped channel assignment on wireless mesh network backbone: A game theoretic approach. IEEE Journal on Selected Areas in Communications, 30(1), 2012, 119-127.

[Frieze, 97] Frieze, A., Jerrum, M.: Improved approximation algorithms for MAX k-cut and MAX bisection. Algorithmica, 18 (1), 1997, 67-81.

[Gupta, 00] Gupta, P., Kumar, P.R.: The capacity of wireless networks. IEEE Transactions on Information Theory, 46 (2), 2000, 388-404.

[Hoang, 08] Hoang, L.N., Nguyen, T.N.: Minimum Interference Channel Assignment for Multicast in Multi-Radio Wireless Mesh Networks. International Conference on Wireless Communications and Mobile Computing (IWCMC'08), 2008, 626-631.

[Liu, 08] Liu, T.H., Liao, W.J.: On Routing in Multichannel Wireless Mesh Networks: Challenges and Solutions. IEEE Network, 22 (1), 2008, 13-18.

[Liu, 10] Liu, Y., Venkatesan, R., Li, C.: Load-aware channel assignment exploiting partially overlapping channels for wireless mesh networks. IEEE Global Telecommunications Conference (Globecom'10), 2010, 1-5.

[Marina, 10] Marina, M.K., Das, S.R., Subramanian, A.P.: A topology control approach for utilizing multiple channels in multi-radio wireless mesh networks. Computer Networks, 54 (2), 2010, 241-256.

[Mesh, 11] IEEE Draft Standard for Information Technology-Telecommunications and information exchange between systems-Local and metropolitan area net-works-Specific requirements-Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications-Amendment 10: Mesh Networking. 2011, 1-361.

[Mishra, 06] Mishra, A., Shrivastava, V., Banerjee, S., Arbaugh, W.: Partially overlapped channels not considered harmful. ACM SIGMETRICS Performance Evaluation Review, 34(1), 2006, 63-74.

[MIT, 12] MIT Roofnet, 2012, http://pdos.csail.mit.edu/roofnet/doku.php.

[Rad, 06] Rad, A. H. M., Wang, V. W. S.: Joint optimal channel assignment and congestion control for multi-channel wireless mesh networks. IEEE International Conference on Communications (ICC'06), 2006,1984-1989.

[Raniwala, 05] Raniwala A, Chiueh T. Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network. In Proceedings of the 24th Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM). Miami, FL, USA, 2005, 2223-2234.

[Sharma, 11] Sharma, P.C., Chaudhari, N.S.: A graph coloring approach for channel assignment in cellular network via propositional satisfiability. International Conference on Emerging Trends in Networks and Computer Communications (ETNCC'11), April 2011, 23-26.

[Si, 10] Si, W.S., Selvakennedy, S., Zomaya, A.Y.: An overview of channel assignment methods for multi-radio multi-channel wireless mesh networks. Journal of Parallel and Distributed Computing, 70 (5), 2010, 505-524.

[Subramanian, 07] Subramanian, A.P., Gupta, H., Das, S.R.: Minimum Interference Channel Assignment in Multi-Radio Wireless Mesh Networks. The IEEE Communications Society Conference on Sensor, Mesh and Ad Hoc Communications and Networks (SECON '07), June 2007, 481-490.

[Subramanian, 08] Subramanian, A.P., Gupta, H., Das, S.R., Jing. C.: Minimum Interference Channel Assignment in Multiradio Wireless Mesh Networks. IEEE Transactions on Mobile Computing, 7 (12), 2008, 1459-1473.

[Zeng, 07] Zeng, G.K., Wang, B., Ding, Y., Xiao, L., Mutka, M.: Multicast Algorithms for Multi-Channel Wireless Mesh Networks. IEEE International Conference on Network Protocols (ICNP'07), Oct. 2007, 1-10.

[Zeng, 10] Zeng, G.K., Wang, B., Ding, Y., et al.: Efficient Multicast Algorithms for Multichannel Wireless Mesh Networks. IEEE Transaction on Parallel and Distributed Systems, 21 (1), 2010, 86-99.

[Zhou, 12] Zhou, K.X., Jia, X.H., Xie, L.M, et al.: Channel assignment for WLAN by considering overlapping channels in SINR interference model. International Conference on Computing, Networking and Communications (ICNC'12), Jan 2012, 1005-1009.