

# General regularities of height-diameter curves in Hungarian oak, sessile oak and Turkey oak high forests

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## Abstract

The height-diameter curves of natural even-aged Hungarian oak, sessile oak and Turkey oak forests in the regions of Staro Oryahovo, Sherba, Tsonevo and Aytos Forestry Districts have been studied. In order to explore the shape of the curves and, in particular, their steepness, the steepness index SI of Douhovnikov (“the method of natural indicators”) was applied. The curves were divided into three groups according to it: flat curves, medium steep curves and steep curves. Both the traditional height-diameter curves and the corresponding relative curves of Tyurin were studied. Consideration of the traditional height-diameter curves gives a certain ground for investigating together the height-diameter curves of Hungarian oak and sessile oak, disregarding the tree species. This inference was confirmed by comparing the relative height-diameter curves for Hungarian oak, sessile oak and Turkey oak with Tyurin’s uniform curve of relative heights. For these comparisons we used the Willcoxon test. A statistically significant great proximity in their shapes was found. As a uniform average curve of relative heights has been established for all tree species, this proximity indicates a possibility of creating a single fan of height-diameter curves for high Hungarian, sessile and Turkey oak.

## Keywords

steepness of curves, Tyurin’s height-diameter curve

## Introduction

It is important to perfect permanently not only the normative-and-reference base of forestry but also the models and tables for evaluating the volume of standing timber. Therefore, it is essential for scientists to improve their knowledge of forest structure.

By comparing the distribution of tree numbers by diameter in undisturbed pure simple even-aged stands, Tyurin (1938) came to the conclusion that the shape of these curves did not depend on tree species, site conditions nor stocking rate. What had a certain effect were age and thinning. This gave him the reason for developing a uniform curve of the percentage distribution of tree numbers and basal areas by diameter. Later, Nedyalkov (1955) confirmed that the shape of the diameter distribution of beech trees in high stands depended on the mean diameter and, respectively, the age. While studying growing stocks of uneven-aged spruce trees, this same author (1967) established different curves of distribution for the particular generations. For the generations at different ages, the distribution curves had a bell shape but the general curve of the whole stand was described with an exponential function.

On the other hand, Dimitrov (1978) found that age and stocking rate did not affect the shape of diameter distribution of mature beech forests. Later, Dimitrov (2003), having investigated different functions of tree-number distribution, established the existence of three distribution types: symmetric (or normal), right asymmetric and left asymmetric. He cited earlier studies by Tishenko (1926), where the latter, after investigating in detail the diameter distributions in Scots-pine, spruce, aspen and birch forests, found out that these distributions did not depend on stocking rate, age, site index (stand-quality level) but only on the mean diameter and the tree species.

Tretyakov (1927) carried out a detailed study of the structures and variation of some dendro-biometric characteristics in even-aged pure stands, uneven-aged mixed stands and other complex stands. He based his study on original data and data taken from Weise, Kunze and others. He established that the forest structure always had a constant nature regardless of stocking rate, age, tree species, growth conditions and stands, both for simple stands and complex ones. This motivated him to formulate the Law of the Uniformity of Stand Structure.

Another important characteristic of the structure of the forest stand is the height-diameter curve. Developing the idea of universal relative curves, Prof. Tyurin proposed a universal height curve expressed in percentages, which he called „normal numbers“ (Tyurin, 1934; Mihov, 2005).

The results of the above-mentioned studies and others, such as the ones by Tonchev (2007), Tsakov et al. (2013), Ferezliev (2013) and, Ferezliev, Tsakov (2017), have served as methodical hint and informational basis for our investigation.

In accordance with this, the height-diameter curves of natural Hungarian-oak, sessile-oak and Turkey-oak high forests at ages from 17 to over 140 were analysed in the present study. Our aims included:

1. To analyse the height-diameter curves of natural Hungarian-oak, sessile-oak and Turkey-oak high forests in terms of steepness.

2. To compare the average relative height-diameter curves for these forests with A. V. Tyurin's "normal numbers", which are applied for all tree species.

Based on 1 and 2, to find a way to create generalized systems of height-diameter curves and models for these and other species to facilitate the determination of volumetric and assortment structure of forest stands.

## Material

The present investigation pertains to even-aged Hungarian-oak, sessile-oak and Turkey-oak high forests in the Balkan Mountains, namely in the territories of Staro Oryahovo, Sherba, Tsonevo and Aytos Forestry Districts. It has been carried out within 90 sample plots, 40 of these in Hungarian oak stands, 48 in sessile oak ones and two in Turkey oak ones. The age of the studied forests was within the range from 12 to 155 years. Their height varied from 6 to 28 metres and their stocking rates were mainly within the range from 70% to 90%.

## Methods

The natural levels of thickness (NLT) after Tyurin (1938) were calculated for each sample plot by dividing the diameters by the mean diameter of the stand. In order to apply the steepness index method (Douhovnikov, 1966), the domains of the experimental curves were standardised by transforming them into the interval  $[0, 1]$ . This was done after (Mihov, 2005) resulting in a scale of 10 values 0.05, 0.15, 0.25, ..., 0.95, representing the diameter.

In order to calculate the relative height-diameter curves of Tyurin, a similar transformation was made for the co-domain, i.e. the height. Relative heights were obtained by dividing them by the maximum height.

The steepness index method is a method to create non-disjoint curve fans, i.e. families of curves that can intersect. In the present work, we use it as a simple criterion for estimating the deviation of experimental curves from an idealized mean curve. According to the values of SI of the height-diameter curves were assigned to three groups, namely: (i)  $SI \leq 0.85$  corresponded to steep curves; (ii) SI within the range of  $0.86 \div 1.09$  corresponded to medium-steep curves and (iii)  $SI \geq 1.1$  - to flat curves.

To assess the influence of tree species, the influence of tree species upon the distribution of investigated height curves by steepness was studied. To do this, two approaches were applied to obtain the idealized curve - in approach I it was obtained separately for each tree species, and in approach II - as the average of all data, i.e. for all three species together.

## Results and Discussion

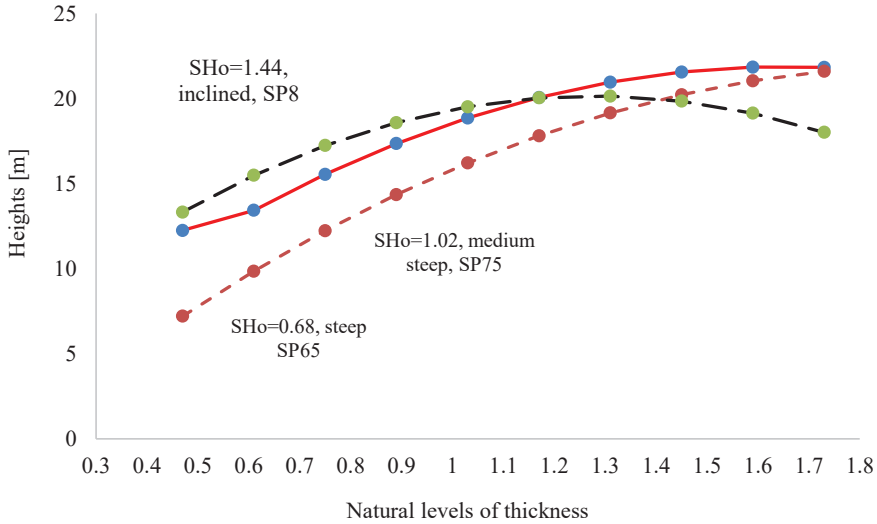
### Height-diameter curve Types according to Curve Steepness

The curves' shapes were investigated by means of the SI which is an indicator of curve's slope. The greater the angle of the curve with the abscissa, the steeper the curve is. The steepness of the curves is related to the difference in heights for two levels of thickness - this difference is greater when the curves are steeper. A steeper curve corresponds to a higher increment in mean height (height increment). Therefore, with all other conditions equal, the volume of a forest will in this case be formed by the thicker and taller trees. Such forests finally have higher volumes and better assortment structures.

SI of relative height-diameter curves (SI) by tree species, have been presented in Table 1.

**Table 1.** Values of the steepness index (SI) of relative height-diameter curves by sample plots, approach I

plot	SI	plot	SI	plot	SI	plot	SI	plot	SI
Hungarian oak		21	0.74	1	1.00	22	0.97	43	1.30
1	0.74	22	0.94	2	0.45	23	0.90	44	1.12
2	0.88	23	1.03	3	0.46	24	1.29	45	0.86
3	0.97	24	0.25	4	0.91	25	1.15	46	0.88
4	0.23	25	3.66	5	0.49	26	1.10	47	0.98
5	1.83	26	0.89	6	1.04	27	0.95	48	1.40
6	0.47	27	1.17	7	1.12	28	1.09	Turkey oak	
7	1.01	28	1.16	8	0.91	29	0.61	1	1.89
8	1.09	29	1.26	9	0.88	30	1.24	2	2.12
9	0.78	30	0.76	10	1.07	31	1.45		
10	0.72	31	0.81	11	0.34	32	1.05		
11	1.01	32	0.66	12	0.94	33	1.15		
12	1.02	33	1.15	13	0.85	34	0.27		
13	0.51	34	1.14	14	0.96	35	1.03		
14	1.12	35	1.28	15	1.03	36	0.96		
15	0.77	36	0.65	16	0.97	37	1.17		
16	1.22	37	0.92	17	1.07	38	1.02		
17	0.56	38	0.88	18	0.81	39	1.08		
18	1.17	39	0.93	19	1.09	40	0.47		
19	1.31	40	1.01	20	1.26	41	1.23		
20	0.95	Sessile oak		21	0.86	42	1.09		



**Figure 1.** Three types of curves according to steepness

SI values after Approach II will not be published here; we shall only discuss the results.

SI varied from 0.23 to 3.66 under approach I. When the general average curve of relative height-diameter curve is used, SI varied from 0.27 to 4.15. The three types of curves according to their steepness determined according to the above ranges of SI, have been illustrated in Figure 1.

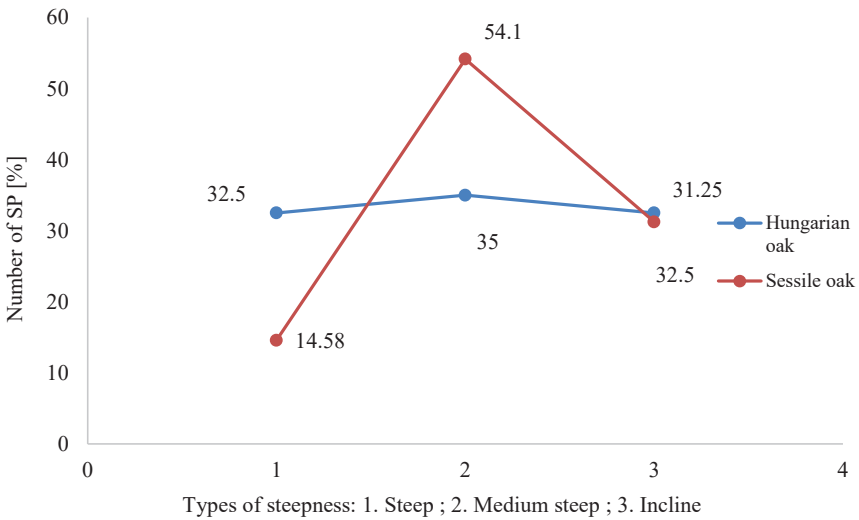
In Figure 1, low values of *SI* (0.68) correspond to a steep curve, high ones ( $SI=1.44$ ) to a flat one, and moderate ones about 1 ( $SI=1.02$ ) to a medium steep one. The steeper the curve, the bigger the difference between two levels of thickness is and the increment in height. Therefore, stands of steeper height curves are supposed to have higher volumes of timber and better assortment structures (Petrin, Markoff, 2015).

### The influence of tree species

Based on the above classification of height curves according to steepness, we estimated their distributions through the steepness index method under both approaches. The difference, as indicated above, was in the average relative height curve used. In the separate investigation separate average curves for the tree species have been used, and in the general one – one average curve. The results of the distribution of the sample plots have been presented in Table 2.

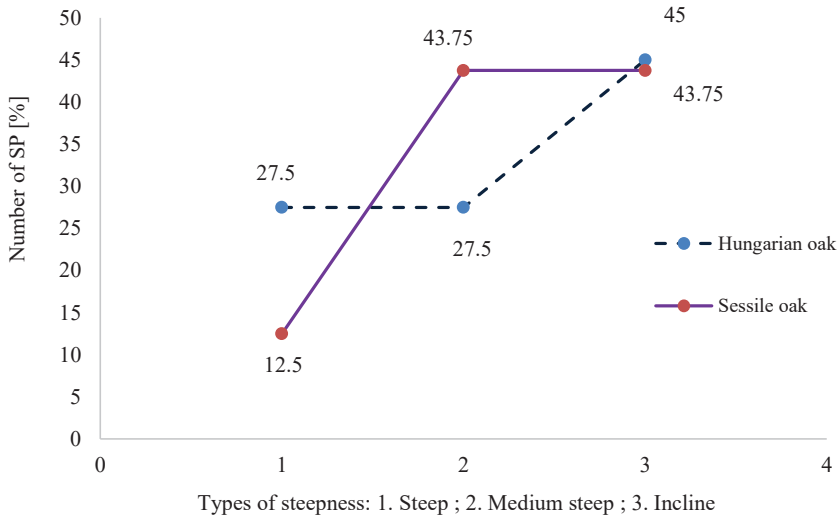
**Table 2.** Distribution of the Sample Plots by steepness of the Height Curves

Tree species	Approach												
	Separate study						In general SP, N	Joint study					
	Type of steepness							Type of steepness					
	Steep		Medium steep		Flat			Steep		Medium steep		Flat	
N	%	N	%	N	%	N	%	N	%	N	%		
Hungarian oak	13	32.5	14	35	13	32.5	40	11	27.5	11	27.5	18	45
Sessile oak	7	14.6	26	54.2	15	31.3	48	6	12.5	21	43.7	21	43.7
Total	20	22.7	40	45.4	28	31.8	88 (100%)	17	19.3	32	36.3	39	44.3



**Figure 2.** Distribution of the numbers of sample by types of steepness and tree species

In the separate study (approach I), both Hungarian and sessile oak were characterised by a similar, though different, distribution of the sample plots according to the types of steepness (see Table 2 and Figure 2). The flat curves were very similar for both the Hungarian and sessile oaks. Medium steep curves dominated within sessile oak forests by 24%, while the steep ones dominated within the Hungarian oak forests by 18%. This leads to the inference that, with all other conditions equal, the increment in height between the particular levels of thickness would be higher within the Hungarian oak forests, which would provide wood of larger assortments and higher quality.



**Figure 3.** Distribution of the numbers of sample plots by types of steepness

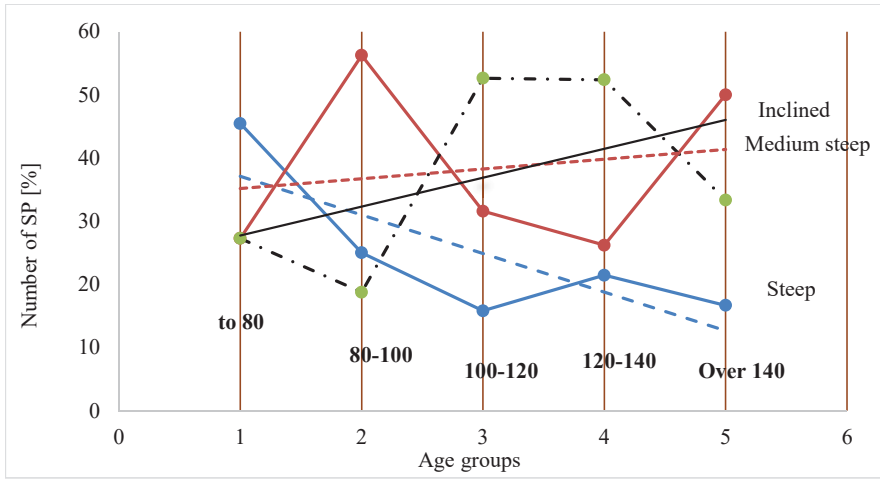
At the same time, Figure 3 reveals the following: flat curves are again very similar for both tree species, while the medium steep ones dominated within sessile oak forests (by 16%) and the steep ones – within Hungarian oak forests (by 15%). Therefore, the overall ratio of the distributions of the sample plots according to the types of steepness has not changed in the joint study of the species.

According to our results, there is a match between the flat curves for Hungarian-oak and sessile-oak forests. There is a similarity only between the ratios of the particular groups of curves according to the types of steepness under both approaches of study. This result was confirmed also by the fact that Hungarian-oak and sessile-oak have almost the same ecological and geographic natural range of distribution (from 0 m a.s.l to 1,000 m a.s.l.) and similar biological characteristics.

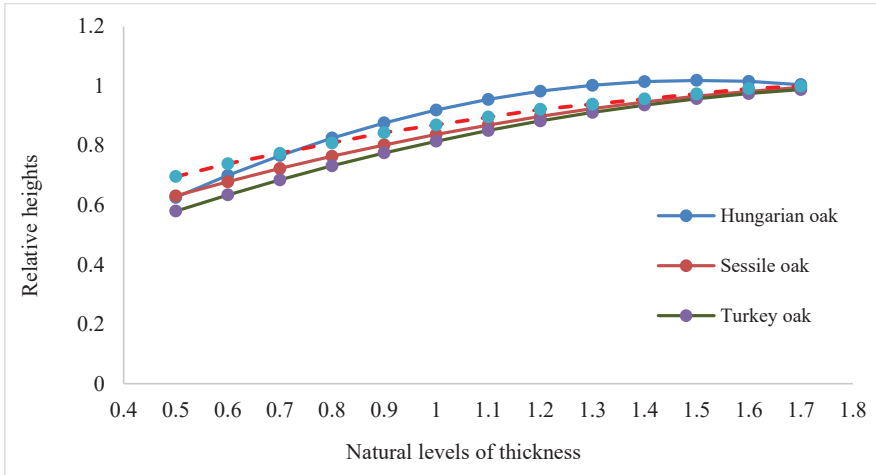
### The influence of age

Further, we have explored the influence of age on the distribution of the sample plots by types of steepness. The following five age groups have been differentiated: up to 80, 81-100, 101-120, 121-140, over 140. The curves presenting the distribution of the sample plots in percentages (%) by age groups and types of steepness, with trend lines, have been presented in Figure 4.

One can see in Figure 4 that clearly-differentiated, non-contradictory trends are present within the three types of steepness. The number of sample plots with flat curves increases with age. On the contrary, the number of steep curves diminishes steadily with age, whereas the distribution curve of the medium steep type of curves



**Figure 4.** Distribution of the numbers of sample plots by types of steepness and age groups



**Figure 5.** Average curves of relative heights

changes only slightly with age. This is only natural as with the increase in age, the energy of tree growth in the stand abates and the increment becomes lower.

Comparing the relative height curves of the oak species with Tyurin’s uniform curve

We have compared in Table 3 our average relative height-diameter curves for Hungarian-oak, sessile-oak and Turkey-oak high forests with Tyurin’s uniform curve of heights that pertains to all tree species

As it is seen from Table 3 and Figure 5, our average curves for the Hungarian, sessile and Turkey oaks were very close to that of Tyurin (1938), which was also confirmed by the calculated statistical parameters in the Wilcoxon test, e.g.



**Table 3.** Average Curves of Relative height-diameter curves (qxav) for Natural High Forests of Hungarian Oak, Sessile oak and Turkey Oak as Compared with Tyurin's Uniform Curve of Normal Numbers

Belonging to the curve	Natural levels of thickness										Wilcoxon test ( $\alpha=0.05$ )						
	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	Stt. Err	t stt.	p	Zero hypothesis
	Average relative height-diameter curves - $q_{kav}$																
Hungarian oak	0.63	0.70	0.77	0.83	0.88	0.92	0.96	0.98	1.00	1.01	1.02	1.02	1.00	19.5	1.05	0.15	assumed
Sessile oak	0.63	0.68	0.72	0.76	0.80	0.84	0.87	0.90	0.92	0.95	0.97	0.98	1.00	19.5	-0.64	0.26	assumed
Turkey oak	0.58	0.63	0.68	0.73	0.78	0.81	0.85	0.88	0.91	0.94	0.96	0.97	0.99	19.5	-0.94	0.17	assumed
Tyurin's uniform average curve	0.69	0.73	0.77	0.81	0.84	0.87	0.89	0.92	0.94	0.96	0.97	0.99	1	-	-	-	-

standard error, t statistics and the parameter  $p$ , which was higher than  $\alpha=0.05$ . This is a very important result that confirms Tyurin's uniform average curve of relative heights. It also suggests the possibility of developing a single fan of height-diameter curves for the natural high forests of Hungarian, sessile oak and Turkey oak.

## Conclusions

Three types of height-structure curves have been identified for the explored natural high forests of Hungarian oak, sessile oak and Turkey oak, and namely: steep, medium steep and flat curves. The low values of steepness indices  $SI \leq 0.85$  corresponded to steep curves; their average values within the range of  $0.86 \leq SI \leq 1.09$  corresponded to the medium steep ones; and their maximum values  $SI \geq 1.1$  – to flat curves.

The separate investigation of the Hungarian oak and the sessile oak has identified a similar, though different, distribution of the sample plots according to the types of steepness was characteristic of each of the species. The slightly flat curves were very similar for the Hungarian and the sessile oaks. The medium steep ones dominated the sampling plots of the sessile oak, while the steep ones dominated the plots with the Hungarian oak. With equal average height, the height increment between the particular levels of thickness would probably be higher for the Hungarian oak and the obtained wood would be of larger assortments and better quality.

The number of sample plots with flat curves increases with age. On the other hand, the number of steep curves diminishes steadily with age, whereas the distribution curve of the medium steep type of curves changes only slightly with age.

The average relative height-diameter curves for natural high forests of Hungarian-oak, sessile-oak and Turkey-oak are very similar to that of Tyurin's uniform curve of relative heights.

The obtained results prove the possibility of creating a single set height-diameter curves for natural high forests of Hungarian oak, sessile oak and Turkey oak, which simplifies modelling of their growing stock and its assortment structure.

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