

Analytical model for determining the leakage albedo component for a direct cylindrical channel passing through the nuclear reactor protective layer*

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Academic editor: Georgy Tikhomirov ♦ Received 29 September 2020 ♦ Accepted 29 January 2021 ♦ Published 21 June 2021

Citation: Kupriyanov KS, Pereverzentsev VV (2021) Analytical model for determining the leakage albedo component for a direct cylindrical channel passing through the nuclear reactor protective layer. Nuclear Energy and Technology 7(2): 91–95. <https://doi.org/10.3897/nucet.7.68941>

Abstract

The task of determining the radiation situation, including neutron and gamma-quantum flux density, radiation spectrum, specific volumetric activity of radioactive gases in the air, etc. behind the protective composition having inhomogeneities, has always been important in matters of radiation safety. One of the ways to solve the problem of determining gamma radiation fluxes was to divide the total ionizing radiation flux into four components: line-of-sight (LOS), leakage, line-of-sight albedo, and leakage albedo, and obtain an analytical solution for each component. The first three components have been studied in detail in relation to simple geometries and there are analytical solutions for them, but there is no such a solution for the last component. The authors of this work have derived an analytical representation for the leakage albedo component, which, in contrast to numerical methods (such as Monte Carlo methods), makes it possible to analyze the effect of inhomogeneities in protective compositions on the radiation environment as well as to quickly obtain estimated values of fluxes and dose rates. Performing a component-by-component comparison, it becomes possible to single out the most significant mechanisms of the dose load formation behind the nuclear reactor protection, to draw conclusions about the effectiveness of design solutions in the protection design and to improve the protection at significantly lower computational costs.

Finally, the authors present calculations for the four components of the total ionizing radiation flux for various parameters of the cylindrical inhomogeneity in the reactor protection. Based on the obtained values, conclusions are made about the importance of taking into account the leakage albedo component in the formation of the radiation situation behind the core vessel.

Keywords

Leakage albedo, inhomogeneities in protective compositions, radiation protection.

Introduction

Inhomogeneities in protective compositions are subdivided into elementary (simple) and complex ones. Ele-

mentary inhomogeneities include those in which the radiation field does not depend on the field of a neighboring inhomogeneity. The study of complex inhomogeneities is a more general problem and, as a rule, does

* Russian text published: *Izvestiya vuzov. Yadernaya Energetika* (ISSN 0204-3327), 2021, n. 1, pp. 133–142.

not have an analytical solution (Nikolaev 1985, Gusev et al. 1989, Memarianfard 2009, Zinchenko et al. 2010, Bukhtoyarova and Semenyak 2015, Radiation Protection and Radiation Safety 2015, Tashlykov et al. 2015). Quite complex engineering problems are solved by numerical (in essence, approximate) methods. However, the results of numerical solutions must be verified. One of the most reliable methods for verifying a numerical solution, and hence a program code, is to compare the calculation results with an analytical solution. At the same time, it is clear that a test problem with an analytical solution should be 'as close as possible' to the original engineering problem to be solved. From this point of view, accumulating the volume of test problems that have analytical solutions is of great practical importance. The objective of this work is to obtain the formula of the leakage albedo component for an elementary inhomogeneity and perform its analysis, which will make it possible to evaluate the influence of the geometric dimensions of the cylindrical channel and the properties of the protective material.

The authors consider an elementary straight cylindrical channel with a diameter of $2a$ passing through the protective layer. On the one hand, there is a plane uniform power source N_0 ; the detection point D is at the outlet from the protection on the channel axis (Fig. 1).

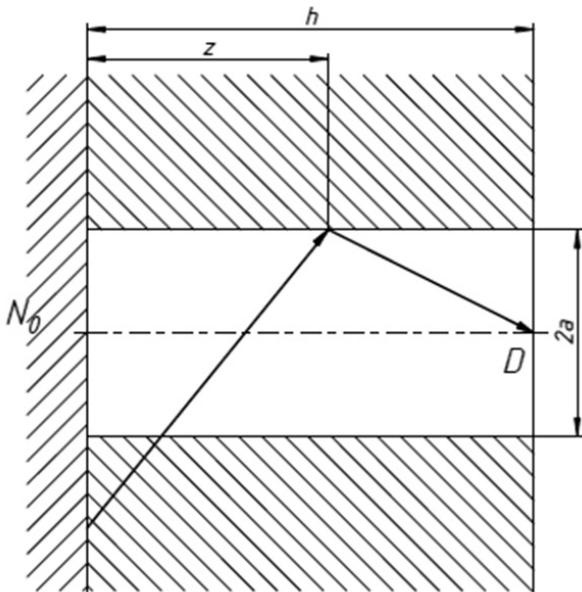


Figure 1. Cylindrical inhomogeneity in the protection

In the figure, the ray shows the formation of the leakage albedo component — the ray leaves the zone outside the channel, goes into the channel region, being physically attenuated, is reflected from the channel wall and hits the detection point D . Let us obtain a solution for this component of the flux. The general law of physical attenuation for ionizing radiation is written as $\exp(-m_0 L)$ (Mashkovich and Kudryavtseva 1995), where the value of m_0 , depending on the type of radiation, is determined differently:

- $m_0 \circ m$ – for gamma quanta ;
- $m_0 \circ [I(E_n > E_n^*)]^{-1}$ – for fast neutrons;
- $m_0 \circ Sa$ – for thermal neutrons.

For simplicity, we shall first consider the plane problem, and then move on to the spatial solution. Let us consider a top view (from the end of the channel) and mark the elements responsible for the formation of the leakage component (Fig. 2). To do this, we select the elementary section dS at radius R and angular coordinate α . To find the flux incident on the point P , we draw a tangent at this point to the channel surface and consider separately the 'parts' of the radiation to the left and right of this tangent.

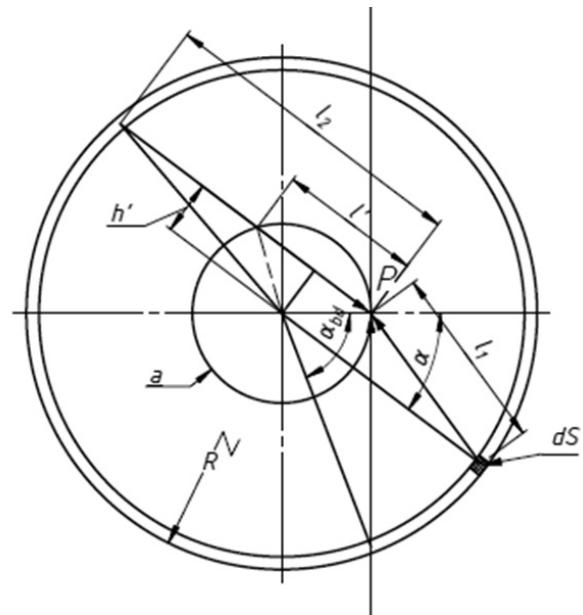


Figure 2. Top view of the inhomogeneity

Let us consider the right section: at radius R , we select the area dS to the right of the tangent. The incident flux is attenuated over the entire section of length l_1 . Since α is the angle between the ray drawn from the center of the circle through the point P and the ray drawn from the center of the circle to the center of the area dS , then for $\alpha \in [0, \alpha_{bd}]$ we can write the expression for the flux:

$$d\Phi = R \frac{N_0 f(\theta)}{l_1^2} dR d\alpha \cdot \exp(-\mu_0 l_1) \quad (1)$$

where $f(\theta)$ is the angular distribution of the source radiation; $l_1 = (R^2 + a^2 - 2Ra \cos(\alpha))^{1/2}$ (by the cosine theorem); $\alpha_{bd} = \arccos(a/R)$.

Assuming that the source is isotropic ($f(\theta) = 1/(2\pi)$), and integrating, we obtain an expression for the flux incident on the channel wall from the part of the ring of thickness dR on the right side (the expression is multiplied by 2, since the flow is summed up and down from the point P):

$$d\Phi_1 = 2N_0 \int_0^{\alpha_{bd}} \frac{R}{2\pi l_1^2} dR d\alpha \cdot \exp(-\mu_0 l_1) \quad (2)$$

Similarly, we can obtain the total incident flux from the part of the ring on the left side. The difference between the right and left sides along the length of the attenuation section (see Fig. 2):

$$d\Phi_2 = 2N_0 \int_{\alpha_{bd}}^{\pi} \frac{R}{2\pi l_2^2} dR d\alpha \cdot \exp(-\mu_0 [l_1 - l']) \quad (3)$$

where $l_2 = l_1 = (R^2 + a^2 - 2Ra \times \cos(a))^{1/2}$ (by the cosine theorem); $l\phi = 2a[1 - \sin^2(a) \times (R/l_2)^2]^{1/2}$. The expression for $l\phi$ is obtained from the relations $h\phi/R = -a \times \sin(a)/l^2$ (the sine theorem for a triangle with side l_2) and $l\phi/2 = (a^2 - (h\phi)^2)^{1/2}$. Further on, equal lengths l_2 and l_1 will be denoted by the symbol l .

Now, we shall turn to the spatial problem (Fig. 3). The figure presents a cylinder with a cut, and the ray shows the formation of the leakage albedo component. The cylinder is assumed to have an infinite radius (to ignore edge effects). When we pass to the spatial problem, the lengths of physical and geometric attenuation, namely the values of l and $[l - l\phi]$, will change. Let us express the new lengths in terms of l , $l\phi$ and z so as not to complicate the formula with new variables.

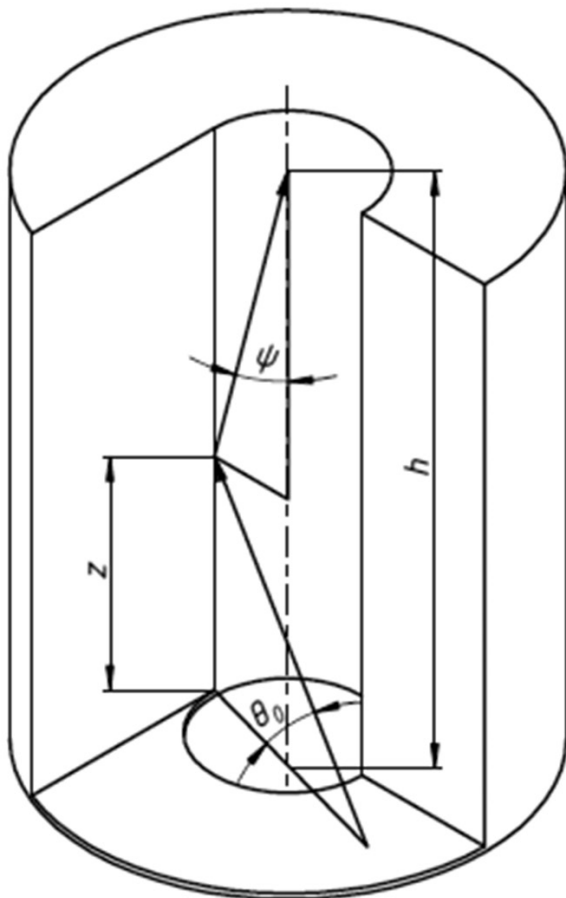


Figure 3. Channel in space

To illustrate how the new geometric attenuation lengths are calculated, Fig. 4, complementing Fig. 3, is used. It shows a triangle of height z and angle q_0 opposite to z . The flux of gamma quanta escaping from the end of the cylinder passes part of the path through the protective material and is thereby physically attenuated along the length $L_{\text{attenuation}}$, and then enters the channel, where only geometric attenuation is present.

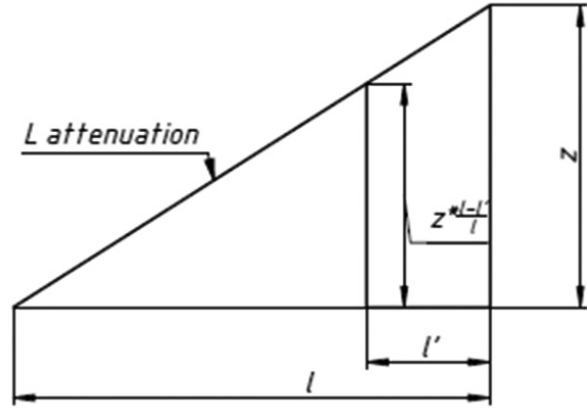


Figure 4. Length of physical attenuation for dF_2

The length of geometric attenuation L is determined by the expression $(l^2 + z^2)^{1/2}$; whereas the expression $[(z(l - l\phi)/l)^2 + (l - l\phi)^2]^{1/2}$ is used to determine the length of physical attenuation.

Then formulas (2) and (3) can be represented as

$$d\Phi_1 = 2N_0 \int_0^{\alpha_{bd}} \frac{R}{2\pi (l^2 + z^2)} dR d\alpha \cdot \exp(-\mu_0 \sqrt{(l^2 + z^2)}) \quad (4)$$

$$d\Phi_2 = 2N_0 \int_{\alpha_{bd}}^{\pi} \frac{R}{2\pi (l^2 + z^2)} dR d\alpha \cdot \exp\left\{-\mu_0 \sqrt{\left[\frac{z(l-l')}{l}\right]^2 + (l-l')^2}\right\} \quad (5)$$

In most practical cases, we can consider the source as an infinite plane and perform integration over the radius with an upper limit equal to infinity. Thus, the total flux incident at the point P from the infinite plane is

$$\Phi_{tot} = \int_a^{\infty} (d\Phi_1 + d\Phi_2) dr$$

To obtain the values of the reflected flux, we shall use the value of the numerical differential albedo: this value depends on the angle of incidence q , the angle of reflection y and the energy of the ionizing radiation flux. The angle of incidence depends on the angle a , therefore, this value must be taken into account even before the first integration over the angle a .

The task is axisymmetric; therefore, integrating the differential of the flux incident onto the lateral surface along a narrow ring, we finally obtain a solution for the leakage albedo component as the sum of two components:

$$F_{\text{leak.alb}} = F_{\text{leak.alb1}} + F_{\text{leak.alb2}};$$

$$\Phi_{\text{leak.alb1}} = \int_0^h \int_a^\infty \int_{\alpha_{\text{bd}}}^\pi \frac{N_0 R}{\pi(l^2 + z^2)} dR d\alpha \times$$

$$\times \exp\left(-\mu_0 \sqrt{l^2 + z^2}\right) \frac{a_n(E_0, \theta_0, \psi)}{(h-z)^2 + a^2} 2\pi \alpha dz$$

$$\Phi_{\text{leak.alb2}} = \int_0^h \int_a^\infty \int_0^{\alpha_{\text{bd}}} \frac{N_0 R}{\pi(l^2 + z^2)} dR d\alpha \times$$

$$\times \exp\left\{-\mu_0 \sqrt{\left[z(l-l')/l\right]^2 + (l-l')^2}\right\} \frac{a_n(E_0, \theta_0, \psi)}{(h-z)^2 + a^2} 2\pi \alpha dz$$

Using the Heaviside function $H(a - a_{\text{bd}})$, we can write the solution in a single integral:

$$\Phi_{\text{leak.alb}} = \int_0^h \int_a^\infty \int_0^\pi \frac{N_0 R}{\pi(l^2 + z^2)} dR d\alpha \times$$

$$\times \frac{\exp\left\{l-l' H(\alpha - \alpha_{\text{bd}})\right\} \sqrt{(z/l)^2 + 1} a_n(E_0, \theta_0, \psi)}{(h-z)^2 + a^2} 2\pi \alpha dz \quad (6)$$

To check the obtained formula, Monte Carlo calculations were performed using SERPENT (a multi-purpose three-dimensional Monte Carlo particle transport code) (Leppanen Jaakko 2015). The calculation was carried out in the one-speed approximation with a uniformly distributed source of isotropic radiation. The geometric model specified in SERPENT is shown in Fig. 5.

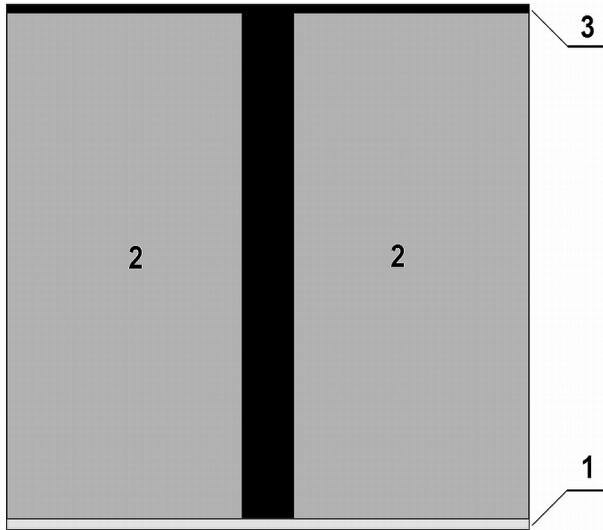


Figure 5. Geometric model in SERPENT: **1** – area in which the source of unit power was specified; **2** – material of the protective composition; **3** - areas without the material

In the model calculation, aluminum was used as a protective material, and the source was set to be monoenergetic with energy of gamma quanta equal to 1.25 MeV. To obtain a flux of gamma quanta on the channel axis at the outlet from the protection, a small finite volume was set in the model, sufficient to register gamma quanta emitted from the source. The results of model calculations, as well as calculations by

Table 1. Results of calculations using the Serpent code and analytical formulas

a , cm	h , cm	SF_{Serpent} , $\text{cm}^{-2}\text{s}^{-1}$	$SF_{\text{analyt.}}$, $\text{cm}^{-2}\text{s}^{-1}$	$e = [(SF_{\text{analyt.}} - SF_{\text{Serpent}}) / SF_{\text{Serpent}}] \times 100$, %
15	135	0.011	0.01	-9.1
15	270	3.501×10^{-3}	2.291×10^{-3}	-34.6
30	135	0.037	0.036	-2.7
30	405	5.252×10^{-3}	3.875×10^{-3}	-26.2
10	200	2.241×10^{-3}	1.968×10^{-3}	-12.2

analytical formulas, are presented in Tab. 1. The values of the physical properties of aluminum $a_n(E_0, q_0, y)$ and $m_0(E_0)$ were taken from (Mashkovich and Kudryavtseva 1995).

Let us now analyze the results obtained.

- Comparing the fluxes obtained analytically and by the Monte Carlo method, one can see that the analytical result always gives lower values.
- Minimum errors are obtained in the case when the inhomogeneity is large and the main contribution to the flux formation is made by the line-of-sight component. If the influence of the leakage components grows, the error increases, which is caused by the violation of the correctness of the assumptions made in the derivation of the analytical formulas.

Let us estimate the contribution of the leakage albedo component to the total ionizing radiation flux density at the detection point. Calculations for a particular case will be presented below.

The monoenergetic ionizing radiation source is ^{60}Co ; the reflective surface is aluminum. The values of $a_n(E_0, q_0, y)$ were taken from (Mashkovich and Kudryavtseva 1995). The surface source rate is $N_0 = 1\text{s}^{-1}$. The formulas for calculating the components of line-of-sight (F_{LOS}), leakage ($F_{\text{leak.}}$) and line-of-sight albedo ($F_{\text{LOS alb.}}$) were taken from (Zolotukhin et al. 1968). The calculation results for the detection point located on the channel axis at height h are shown in Tab. 2.

Table 2. Calculations of individual components of the ionizing radiation flux density

a , cm	h , cm	m_0 , cm^{-1}	F_{LOS} , $\text{cm}^{-2}\text{s}^{-1}$	$F_{\text{leak.}}$, $\text{cm}^{-2}\text{s}^{-1}$	$F_{\text{LOS alb.}}$, $\text{cm}^{-2}\text{s}^{-1}$	$F_{\text{leak. alb.}}$, $\text{cm}^{-2}\text{s}^{-1}$	$F_{\text{leak. alb.}} / SF$
15	350	0.3	9.18×10^{-4}	3.93×10^{-6}	1.786×10^{-4}	7.451×10^{-5}	0.063
5	350	0.3	1.02×10^{-4}	1.087×10^{-9}	6.795×10^{-6}	1.462×10^{-5}	0.118
5	115	0.3	9.443×10^{-4}	5.759×10^{-5}	1.856×10^{-4}	2.44×10^{-4}	0.17
30	350	0.3	3.66×10^{-3}	5.341×10^{-5}	1.17×10^{-3}	2.043×10^{-4}	0.04
15	115	0.3	8.435×10^{-3}	5.265×10^{-4}	3.104×10^{-3}	9.932×10^{-4}	0.076
15	115	3	8.435×10^{-3}	8.383×10^{-6}	3.104×10^{-3}	9.452×10^{-5}	0.0081
5	350	3	1.02×10^{-4}	0	6.795×10^{-6}	1.377×10^{-6}	0.012

Let us analyze the results presented in this table:

- For substances with high m_0 the contribution of the leakage components is orders of magnitude less than that of the line-of-sight components.
- As height h increases, the contribution of the leakage albedo can both decrease and increase.

- With a decreasing in size, the value of the line-of-sight flux decreases more intensively than the value of the leakage albedo flux, which increases the contribution of the latter to the total flux.

Therefore, it is important to take into account the leakage albedo in the case of small channels in protections with ‘low’ values of m_0 . For a particular case from the given example, the leakage albedo can be 17% of the total flux.

Conclusion

Despite the rapid development of numerical methods in the calculation of the radiation environment, analytical solutions still find their application in the initial estimates of radiation fields, in the study of the dependences of the obtained fluxes of ionizing radiation, as well as for verification of software systems (Sobol 1973, RSICC 2000, Robert

and Casella 2004, Gomin 2006, Leppanen Jaakko 2015). In ionizing radiation transfer processes, especially in structures of complex geometry (in particular, in the presence of inhomogeneities) with sharp changes in the quantitative characteristics of the interaction of radiation with matter, numerical solutions in most cases do not provide a clear (physically transparent) understanding of the laws governing the formation of ionizing radiation field functionals. Therefore, from the point of view of the formation of clear and correct ideas about the laws governing the propagation of ionizing radiation, the possibility of obtaining analytical solutions is of major educational and methodological significance. This circumstance is extremely important for training specialists in calculating protections against ionizing radiation. The authors of this work obtained and analyzed an analytical solution for the leakage albedo component. It is shown that this component can be important for certain channel parameters and should be taken into account when calculating the radiation situation.

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