NUCET NUCLEAR ENERGY AND TECHNOLOGY

Nuclear Energy and Technology 9(3): 171–175
DOI 10.3897/nucet.9.112165

Research Article

The effect of the importance function resolution on the accuracy of calculating the functionals of the neutron kinetics in water critical assemblies by Monte Carlo method*

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Academic editor: Georgy Tikhomirov ♦ **Received** 14 January 2023 ♦ **Accepted** 12 May 2023 ♦ **Published** 20 October 2023

Citation: Arkhangelsky DM, Daichenkova YuS, Kalugin MA, Oleynik DS, Shkarovsky DA (2023) The effect of the importance function resolution on the accuracy of calculating the functionals of the neutron kinetics in water critical assemblies by Monte Carlo method. Nuclear Energy and Technology 9(3): 171–175. https://doi.org/10.3897/nucet.9.112165

Abstract

The paper considers a computational study of the importance function effect on the accuracy of calculating the effective fraction of delayed neutrons, β_{eff}, and generation time of instantaneous neutrons using the MCU Monte Carlo code based on the example of three criticality experiments from the ICSBEP handbook. In the MCU code, the importance function has a piecewise constant form: the computational model is broken down into a finite number of registration objects, and the neutron importance is calculated in each. The obtained importance values are used then to calculate the kinetic functionals due to which the calculation accuracy for the latter depends on the resolution. Three types of the importance function spatial partition (axial, radial, combined) have been studied. The numerical simulation results have shown that the axial component of the neutron importance function in all experiments has practically no effect on the calculation accuracy for β_{eff} and Λ: the difference between the obtained values is less than 1%. The radial component has a notable effect (of up to 15.9%) on the Λ calculation accuracy while having almost no effect on the β_{eff} estimate. Using combined partition, as compared with radial partition, improves the calculation accuracy insignificantly (< 1%).

Keywords

Monte Carlo method, MCU, kinetic functionals, importance function, generation time, delayed neutrons

Introduction

Presently, the Monte Carlo method has been increasingly often used for solving transport problems in nuclear reactor design (Sobol 1973). The evolution of parallel computing technologies has made it possible to consider complex 3D systems and achieve a high precision level of calculations (Voevodin and Voevodin 2002; Andrews 2003).

Critical in this respect is to improve the accuracy of estimating the neutron kinetics functionals: effective fraction of delayed neutrons, β_{eff}, and prompt neutron generation time, Λ (Lewins 1960; Kipin 1967; Hetrick 1975).

* Russian text published: Izvestiya vuzov. Yadernaya Energetika (ISSN 0204-3327), 2023, n. 2, pp. 5–14.

These functionals were calculated earlier in the MCU code in the assumption that the importance function is a constant in the entire phase volume. For the MCU code (Kalugin et al. 2015; Gurevich et al. 2019), the capability is currently developed for taking into account the neutron importance function in β_{eff} and Λ computations (Hendricks et al. 2005; Brun et al. 2015), offered in some other codes (TRIPOLI, MCNP (Gurevich et al. 2009; Kalugin et al. 2011). In this study, the importance function is presented as discretely partitioned: the computational model is broken down into a finite number of registration objects, the neutron generation density and its adjoint function being computed in each.

This paper looks into the effect of the importance function resolution on the accuracy of β_{eff} and Λ estimations.

As part of the study, experiments were considered involving low-enriched uranium in composite systems with a thermal spectrum conducted on light-water critical assemblies, ZR-6 (LCT-015) and Stend P (LCT-053 and LCT-085) from the ICSBEP Handbook (NEA 2002).

The paper presents a brief description of the algorithms and methods offered by the MCU code, a description of computational models, and an analysis of the results obtained.

Calculation procedures

The MCU code calculates Λ and β_{eff} based on the representation of these as bilinear functionals of neutron importance

$$\Lambda = (\psi^{+}, \tau(\mathbf{F} + \mathbf{B})\psi) / (\psi^{+}, (\mathbf{F} + \mathbf{B})\psi), \quad (1)$$

$$\beta_{eff} = (\psi^{+}, \mathbf{B}\psi) / (\psi^{+}, (\mathbf{F} + \mathbf{B})\psi), \quad (2)$$

where $\mathbf{F}$ is the prompt neutron generation operator, $\mathbf{B}$ is the delayed neutron generation operator, τ is the neutron lifetime, ψ is the neutron generation density, and ψ^{+} is the generation density adjoint function which has the meaning of neutron importance (Zolotukhin and Majorov 1984; Brown et al. 2011). Therefore, estimation of functionals requires computing the neutron generation density and its adjoint function.

However, it is difficult to obtain both these functions in a continuous form using the Monte Carlo method. The computational model is therefore broken down into a finite number of registration objects, that is, a set of points in a phase space in which the required functions are registered. Thus, a conditionally critical problem can be presented in a matrix form

$$k_{eff}\psi = \mathbf{T}\psi, \quad (3)$$

where ψ is the vector the i^{th} component of which fits the neutron generation density in the respective object, and $\mathbf{T} = \mathbf{F} + \mathbf{B}$ is the matrix the T_{ij} component of which has the meaning of the average number of neutrons which appear in object i as a result of the nuclide fission by the neutron born in object j (Oleynik 2005; Brown et al. 2013).

The neutron importance function can be obtained by solving the equation adjoint with (3)

$$k_{eff}\psi^{+} = \mathbf{T}^{\mathrm{T}}\psi^{+}, \quad (4)$$

where the transposed matrix $\mathbf{T}^{\mathrm{T}}$ has the meaning of an adjoint operator.

Therefore, the matrix of fissions, $\mathbf{T}$, obtained as a result of Monte Carlo simulations, is transposed and used to solve equation (4). This results in vector ψ^{+}, a neutron importance function partitioned by registration objects.

The obtained values are used then to obtain estimates (1) and (2) that depend on the importance function resolution.

The evaluation of the new methodology undertaken in (Gurevich and Shkarovsky 2012) has shown it to agree with the experimental data.

Description of computational models

This paper deals with light-water critical facilities, ZR-6 (LCT-015) and Stend P (LCT-053 and LCT-085) (see the ICSBEP Handbook).

The Stend P experimental facility represents a triangular lattice of fuel rods submerged in a water-filled tank of stainless steel. Light water is used as the moderator. The facility's criticality is controlled through the selection of the moderator level. The fuel rod spacing in the triangular lattice is 1.27 cm. There are bottom and top support grids to retain the rods in their required positions. Uranium dioxide, UO_2, is used as the fuel composition. The fuel enrichment in the LCT-053 and LCT-085 facility versions under consideration is 4.4% and 6.5% respectively.

The Stend P computational model's vertical and horizontal cross-sections are shown in Figs 1 and 2.

Similarly to the Stend P, the ZR-6 zero-power experimental facility consists of an assembly of fuel rods sub-

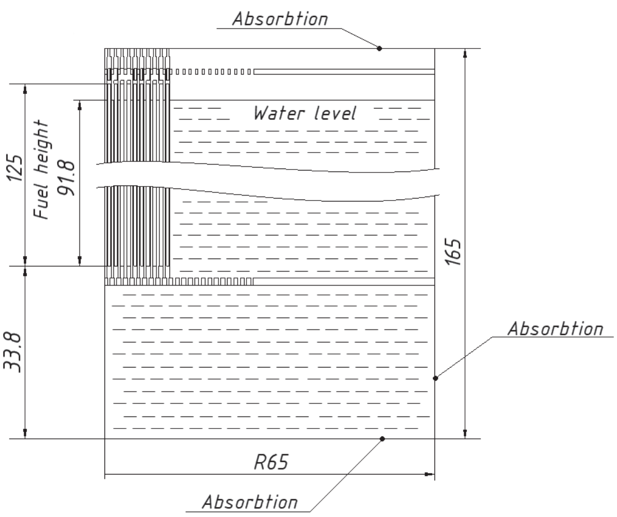

Figure 1. Vertical cross-section of the Stend P computational model.

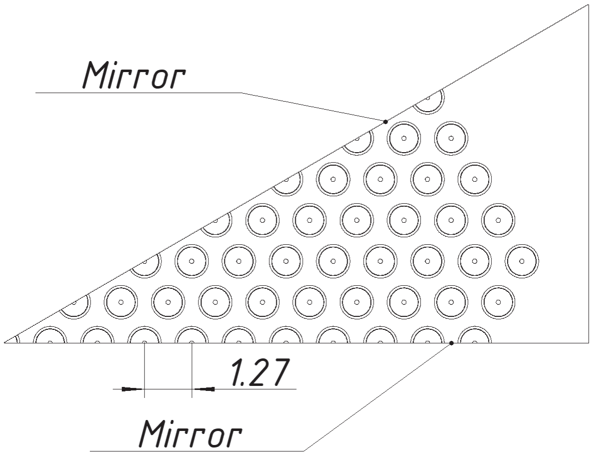

Figure 2. Horizontal cross-section of the Stend P computational model.

merged in a stainless-steel tank. Light water with a boric acid addition (5.8 g/l) is used as the moderator. The triangular fuel lattice spacing is 1.27 cm, and the fuel is UO2 with an enrichment of 4.4 wt. %.

The ZR-6 computational model's vertical and horizontal cross-sections are shown in Figs 3 and 4.

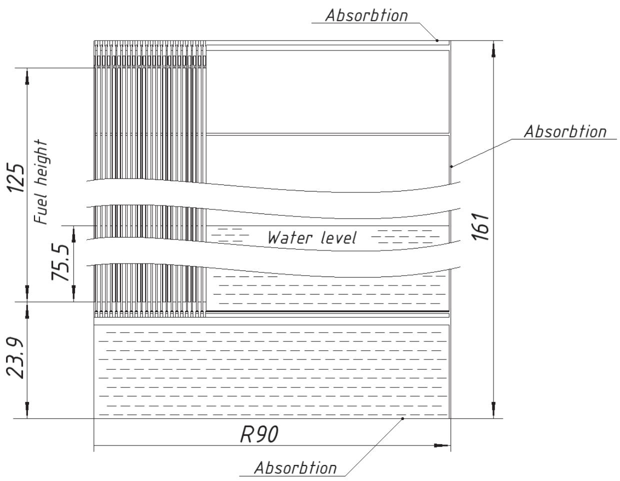

Figure 3. Vertical cross-section of the ZR-6 computational model.

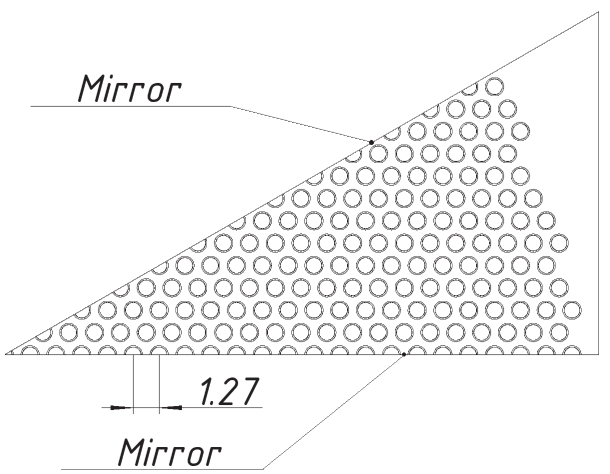

Figure 4. Horizontal cross-section of the ZR-6 computational model.

As part of the study, different methods were used to partition the fuel columns into registration objects (Mihajlov and Rogazinskij 2002; Gurevich and Shkarovsky 2018). Axial partition is based on identifying ten layers along the fuel column height. There is a unique registration object for each fuel column in the radial partition. Combined partition represents a combination of radial and axial partitions. The symmetry angle for all of the presented models is 30 degrees.

Analysis of calculation results

Let us consider the results of calculating β_{eff} and Λ with taking into account and without taking into account the importance function with different resolutions of the neutron importance function.

The calculation results from the LCT-085, LCT-053 and LCT-015 experiments, as well as the value of the estimate correction using the neutron importance are presented in Tables 1 through 3. Figs 5 and 6 present diagrams of the normalized radial and axial dependences of the neutron generation density and the importance function for all experiments under consideration.

Table 1. LCT-085 experiment calculation results, case 13

Partition type	β_{eff}		Λ	
	Value, pcm	Δ, %	Value, 10^{-5} s	Δ, %
Importance not taken into account	815.6	–	3.1	–
Axial	816.2	-0.1	3.1	0.1
Radial	822.1	-0.8	2.7	15.7
Combined	822.4	-0.9	2.7	15.9

Table 2. LCT-053 experiment calculation results, case 11

Partition type	β_{eff}		Λ	
	Value, pcm	Δ, %	Value, 10^{-5} s	Δ, %
Importance not taken into account	804.3	–	3.3	–
Axial	806.6	-0.3	3.3	0.6
Radial	810.4	-0.8	2.9	14.5
Combined	812.7	-1.0	2.8	15.3

Table 3. LCT-015 experiment calculation results, 163/161

Partition type	β_{eff}		Λ	
	Value, pcm	Δ, %	Value, 10^{-5} s	Δ, %
Importance not taken into account	778	–	2.72	–
Axial	783	-0.6	2.60	2.0
Radial	783	-0.6	2.46	10.3
Combined	788	-1.2	2.44	11.5

As shown by the results, the following regularities can be identified in all experiments: the contribution from the radial partition of the importance function is larger than that from the axial partition. In particular, when Λ is calculated, taking into account the importance function has little effect on the β_{eff} computation accuracy. To explain the obtained results, one needs to consider the radial and axial dependence diagrams for the generation density and the neutron importance.

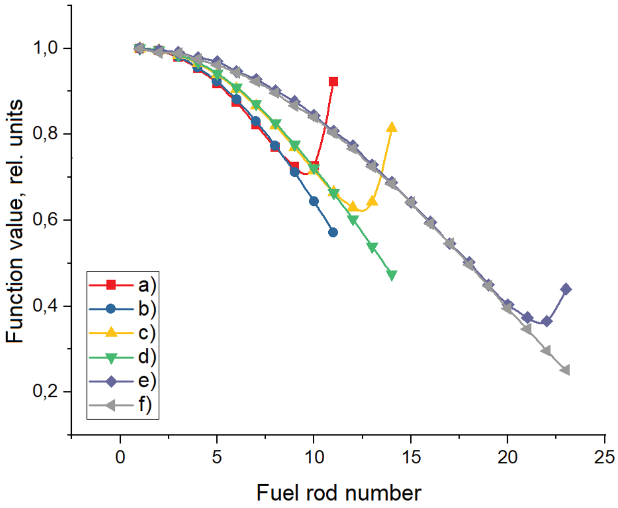

Figure 5. Radial components of neutron generation density and neutron importance: a) – generation density in LCT-085; b) – importance in LCT-085; c) – generation density in LCT-053; d) – importance in LCT-053; e) – generation density in LCT-015; f) – importance in LCT-015.

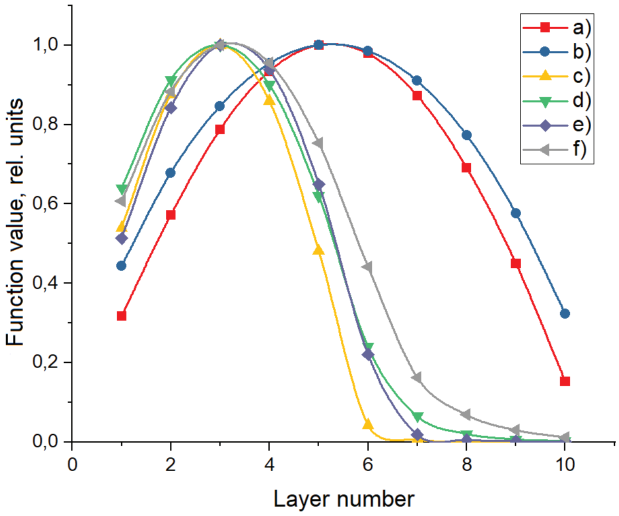

Figure 6. Axial components of neutron generation density and neutron importance: a) – generation density in LCT-085; b) – importance in LCT-085; c) – generation density in LCT-053; d) – importance in LCT-053; e) – generation density in LCT-015; f) – importance in LCT-015.

All radial distributions show an abrupt jump in the neutron generation density on the assembly periphery which is the result of the water-uranium ratio growth on the reflector boundary. The neutrons born in the high importance region can get into the reflector and, after they return, will be highly likely absorbed in the low importance region in the external fuel rod row. To obtain the reliable value of Λ, the long lifetime of such neutrons needs to be included in the final estimate with a weight (importance function) since the secondary neutrons they produce have a small importance due to a high probability of leakage. The increase in the ratio between the neutron generation density on the periphery and in the assembly center leads to an increased effect from taking into account the importance (Mihajlov 1987). Therefore, taking into account the neutron importance function allows one to adjust the prompt neutron generation time estimate up to 16%.

The monotonies of the generation density and neutron importance axial distributions coincide in the entire interval due to which taking into account the importance function does not make a major contribution to the Λ estimate adjustment (up to 2%).

As compared with radial partition, combined partition leads to a slightly improved accuracy of estimates (≈ 1%) as the result of the minor radial partition effect.

Noteworthy, taking into account the importance function has little effect on the accuracy of calculating β_{eff} (≈ 1%) as the result of rather a soft spectrum of the system. The difference in the energy of prompt and delayed neutrons levels off rapidly due to good moderating properties of the medium.

Conclusions

The paper describes in brief the algorithms offered in the MCU code for taking into account the importance function when calculating the effective fraction of delayed neutrons and the prompt neutron generation time.

The effect of the importance function resolution on the accuracy of computing the functionals under consideration was investigated using three partition types (axial, radial, combined).

An analysis of the calculation results has shown that radial partition contributes decisively to the improved accuracy of estimating the prompt neutron generation time (up to 16%) as the result of the neutron generation density jump on the assembly periphery. Peripheral neutrons of minor importance are born largely by the neutrons that have returned from the reflector and have a long lifetime. To obtain correct estimates, the lifetime of these needs to be included in the final estimate with a weight in the form of the importance function at the absorption point.

Since the functions under consideration have similar monotonies, axial partition does not contribute greatly to the improved accuracy of estimates, due to which combined distribution corrects the estimates, this correction being nearly identical to that for radial partition.

Taking into account the importance function has little effect on the accuracy of calculating the effective fraction of delayed neutrons (≈ 1%) β_{eff} in all options due to the good moderating properties of the system: the difference in energy between prompt and delayed neutrons levels off rapidly.

It is therefore required to increase the neutron importance resolution in the peripheral regions in which the neutron spectrum changes abruptly.

Acknowledgement

The results presented in the paper have been obtained using the computational resources from the Joint Computing Cluster of NRC Kurchatov Institute, http://computing.nrcki.ru/.

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