On the concept of “effective delayed neutron fraction”*

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Abstract

The article considers methodological issues related to the conceptual and terminological apparatus of the dynamics of nuclear reactors. Based on an elementary analysis of the standard point reactor kinetics equations, the author shows that it is necessary to clarify the physical meaning of the parameter β included in the equations, which is traditionally interpreted as the “effective delayed neutrons fraction” (EDNF). It follows directly from the kinetics equations that the parameter β, which appears in these equations as the EDNF, is, from the point of view of the neutron balance, the fraction of prompt neutrons consumed for the generation of delayed neutron precursors (DNPs), and, from the point of view of the DNP balance, the DNP yield per prompt neutron in a single fission event. With these interpretations taken into account, the role of the β parameter is considered in situations related with its adjustment by multiplying it by the “delayed neutron efficiency factor” and with the establishment of the actual fractions of prompt and delayed neutrons. In particular, it is shown that:

- the statement “if the delayed neutron fraction is β, then the prompt neutron fraction is equal to 1 – β”, used in the problems of analyzing the nuclear reactor dynamics as a starting position, cannot be considered applicable to any reactor conditions;
- an increase in the β parameter by multiplying it by the “delayed neutron efficiency factor” leads, contrary to traditional interpretations, not to an increase but to a decrease in neutron reproduction in a supercritical reactor.

The proposed clarifications are appropriate both in terms of more adequate descriptions of processes in nuclear reactors and in relation to the formulations of nuclear safety requirements.

Keywords

Nuclear reactor dynamics, effective delayed neutron fraction

Introduction

The “effective delayed neutrons fraction” (EDNF) is one of the key concepts in the physics of nuclear reactors. The corresponding parameter β largely determines the nuclear reactor dynamics as a measure of reactivity and an expression of “prompt criticality”.


and geometry of the nuclear reactor, the EDNF turns out to be a variable (Toshinsky and Bulavin 1967).

However, a detailed consideration of the equations of nuclear reactor dynamics reveals that the parameter $\beta$, which appears in these equations as the EDNF, actually has the meaning of the delayed neutron precursor (DNP) yield per one prompt neutron in a single fission event. And, therefore, it must be a constant. This collision requires a more careful consideration of the role of the parameter $\beta$ in situations related with its correction by multiplying it by the “efficiency factor” and with the establishment of the actual fractions of prompt and delayed neutrons.

Obviously, it is necessary to distinguish between the fractions in single fission events, in reactor steady states and in transient processes. Of interest are both the fractions of the current number of delayed and prompt neutrons relative to the total volume of the neutron population, and the fractions of generation and loss of these particles, i.e., the ratio of the process rates. An elementary analysis of the nuclear reactor dynamics equations (both point and distributed ones) shows that these quantities, as applied to delayed neutrons, are by no means always expressed in terms of the parameter $\beta$. Therefore, the historically formed interpretation of the parameter $\beta$ as the “effective delayed neutrons fraction” requires both substantive and terminological clarifications.

**Parameter $\beta$ in the nuclear reactor dynamics equations**

In the equations of nuclear reactor point dynamics

$$\frac{dn}{dt} = (\rho - \beta)/\Lambda n + \sum_{j=1}^{n} \lambda c_j + Q \quad (1)$$

$$\frac{dc_j}{dt} = -\lambda c_j + (\beta/\Lambda) n, j = 1, J \quad (2)$$

(where all the notations are commonly-accepted: $n$ is the size of the neutron population; $c_j$ is the number of the $j$-th group of the neutron neutrons; $\rho$ is the reactivity; $\Lambda, J$ are the generation time and lifetime of the neutrons; $\beta$, $\lambda$ are the group parameters of the neutrons) describe the reproduction of the neutron population as a balance of the process rates. In this case, the terms are obviously interpreted as follows. The expression

$$v_{\text{pm}} = (\rho/\Lambda) n = (1/\Lambda - 1/J) n$$

is the difference between the generation rate of prompt neutrons $g_{\text{pm}}$ and the loss rate of neutrons $v_{\text{pm}}$ of any origin (prompt and delayed) in the processes of leakage and unproductive absorption:

$$v_{\text{pm}} = n/\Lambda \text{ and } v_{\text{pm}}^l = n/l \quad (3)$$

We will call the value $v_{\text{pm}}$ the prompt neutron reproduction rate in order to distinguish it from the reproduction rate of the population as a whole $dn/dt$.

According to Eqs. (2), the term

$$v_{\text{pm}} = (\beta / \Lambda) n$$

is the total rate of generation of delayed neutron precursors or, in relation to Eq. (1), the rate of consumption of available neutrons per DNP generation. Therefore, the parameter $h = \beta/\Lambda$ has the meaning of the relative rate, i.e., the share of consumption (per unit time) of these neutrons per DNP generation, since

$$h = (\beta/\Lambda) n = n v_{\text{pm}} / n.$$ 

Accordingly, the parameter $\beta = \sum \beta_j$ is the share of prompt neutron consumption per DNP generation, i.e., the DNP yield per one prompt neutron:

$$\beta = v_{\text{pm}} / (n / \Lambda) = v_{\text{pm}} / v_{\text{pm}} = \beta_{\text{pm}}, \quad (4)$$

Thus, the parameter $\beta$ should be considered as a constant that characterizes a single fission event and relates the number of DNP's per fission

$$v_{\text{pm}} = v_{\text{pm}}^l / v_{\text{pm}} \times (v_{\text{pm}} / v_{\text{pm}}) = \beta / v_{\text{pm}} \quad (5)$$

with the corresponding number of prompt neutrons $v_{\text{pm}}$. It is in this guise that the parameter $\beta$ appears in the first works on the nuclear reactor neutron dynamics (Glasston and Edlund 1954).

The interpretation of the parameter $\beta$ in Eq. (1), which took root later in many works as an “effective quantity”, that in some way specifies the delayed neutrons fraction in the total yield per fission, is erroneous both in terms of content and terminology.

Firstly, the parameter $\beta$ has the meaning of a “fraction” (as a characteristic of some part of a set that is homogeneous in terms of a certain attribute) when it characterizes the prompt neutron consumption per DNP generation in Eq. (1). But in Eqs. (2), the parameter $\beta$, as the DNP yield per one prompt neutron, can no longer, strictly speaking, be called a “fraction”.

Secondly, although the decay of the precursor nucleus gives one delayed neutron, i.e., $d_{\text{pm}} = v_{\text{pm}}$, but the fraction of delayed neutrons in the total yield per fission $\beta_{\text{pm}}$ is not equal to the value $\beta = \beta_{\text{pm}}^l$ appearing in Eqs. (1), (2):

$$\beta_{\text{pm}} = v_{\text{pm}} \times (v_{\text{pm}} + v_{\text{pm}}) \neq \beta = v_{\text{pm}} / v_{\text{pm}} \quad (6)$$

Of course, it is possible to use the parameter $\beta_{\text{pm}}$ instead of the parameter $\beta$ according to the obvious relation $\beta = \beta_{\text{pm}} / (1 - \beta_{\text{pm}})$. But then the corresponding coefficients in Eqs. (1), (2) should be changed.

Distributed neutron transport models are structurally similar to Eqs. (1), (2). In particular, the generation of prompt and delayed neutrons is described by the following terms (see, for example, (Bell and Glesson 1974, Bahman 2019)):

$$\chi_j(E) \int_{E_{\text{min}}}^{E_{\text{max}}} (1 - \beta (r, E')) \tau (r, E') \Sigma d \Omega dE + \sum_j \chi_{\text{pm}} (E) \lambda_j \chi_j$$

(7)
However, it is important to note that the parameter $v(r, E')$ here should be interpreted as the prompt neutron yield per fission. Then the meaning of the parameter $\beta(r, E')$ will be similar to that defined by Eqs. (4), (5). If, however, as is sometimes done (Halimonchuk 2008), the parameter $v(r, E')$ is understood as the total yield, including delayed neutrons, then the interpretation $[\beta(r, E')] = \frac{v_{p}^{DNP}}{v_{p}^{DNP} + v_{n}^{DNP}}$ will lead to a disbalance of rates. In this case, the first term in expression (7) is only the prompt neutron generation rate, and the model does not include the neutron consumption rate per DNP generation. As applied to equation (1), this means that we neglect the term $\frac{dC}{dt} = \frac{r(t)\eta(t)}{h_{0}}$ (8).

That is, in the case of criticality ($r = 0$), a stationary regime is not provided. This indicates a disbalance.

### About “efficiency” of parameter $\beta$

The parameter $\beta$ can be omitted explicitly and only the rate constant $h_{0} = \beta/\Lambda$ can be used, i.e., the fraction of the neutron population that goes into the DNP generation per unit time. This is especially convenient if system (1), (2) is transformed into an integral equation

$$v(t) = r(t)\eta(t) - h_{0}\int_{0}^{t} h(t - \tau) v(\tau) d\tau + Q(t).$$

Here, the neutron population reproduction rate $v(t) = dn/dt$, reactivity in the $\Lambda$-scale $r(t) = \rho/\Lambda$ has the meaning of the relative prompt neutron reproduction rate $r(t) = r_{n}(t) = \frac{\nu_{n}(t)}{\eta(t)}$, normalized DNP reproduction function

$$h(t - \tau) = \sum_{j} a_{j} \exp\left(-\lambda_{j} (t - \tau)\right), a_{j} = \beta / \beta_{j},$$

DNP reproduction integral

$$Y(t) = h_{0}\int_{0}^{t} h(t - \tau)v(\tau) d\tau = \frac{dC}{dt}, C = \sum c_{j},$$

is equal to the current total reproduction rate of the DNP

$$\frac{dC}{dt} = h_{0}n - \sum \lambda_{j} c_{j}.$$

Equation (8) written as

$$v(t) = r(t)\eta(t) - \frac{dC}{dt} + Q(t)$$

shows that the model under consideration describes the dynamics of the neutron population with the reflection of only two processes: (1) the prompt neutron reproduction at a rate of $r(t)\eta(t)$ and (2) the DNP reproduction at a rate of $\frac{dC}{dt}$. With this level of detail, there is no need to introduce the “effective” parameter $\beta$ into equation (9), which determines the DNP reproduction rate.

This is due to the fact that equations (1), (2), (8) describe only the change in the neutron population. In this case, to reflect the increased contribution of delayed neutrons, it would be necessary to correct the term $\sum \lambda_{j} c_{j}$, the delayed neutron generation rate. However, in fact, models (1), (2), (8) do not require such refinement. Firstly, these models operate on population size as such, without any qualitative subdivision of neutrons. Secondly, the balance of replenishment-consumption rates the population is determined only by the current values of the generation time $\Lambda$ and the lifetime $l$. Only these parameters reflect the geometry and material composition of a particular reactor, determining the importance of neutrons of any origin, including delayed ones. The delayed neutrons simply act as an additional source with an “actual” generation rate $\Sigma \lambda_{j} c_{j}$.

The parameters $\Lambda$ and $l$ completely determine the reactor multiplying properties, since they express the criticality condition $r = 1 / \Lambda - 1 / l = 0$. This condition requires the equality of the direct ($\beta/\Lambda$) and inverse $\Sigma \lambda_{j} c_{j}$ rates of the two-way reaction “neutrons $\leftrightarrow$ DNPs”. Obviously, equality is invariant to multiplication by the efficiency factor, i.e., within the framework of the model under consideration, the nuclear reactor criticality does not depend on the importance of delayed neutrons. In other words, only the replenishment of the population with delayed neutrons is taken into account here, and their importance is indirectly reflected in the parameters $\Lambda$ and $l$. Therefore, the parameter $\beta$ should be treated precisely as a constant defined by relations (4), (5). It would be inappropriate to correct the parameter $\beta$ in any way by multiplying it by the delayed neutron efficiency factor and then interpret the resulting value as “effective”.

Let us illustrate this with two examples.

After zero reactivity is established, equation (8), which in this case takes the form $v(t) = -\frac{dC}{dt}$ (at $Q = 0$), should describe the actually observed decay of DNPs with the corresponding fractions $\sum \lambda_{j} c_{j} = \beta$. The use of the “effective” parameter $\beta$ will distort the real picture of the decay.

Further, in the case of $v(t) > 0$, $r(t) = 0$, the first term in Eq. (8) has the meaning of the neutron population generation rate, and the second term denotes the loss rate of neutrons that go to the DNP generation. In this case, the second term is $dC/dt > 0$. Therefore, according to Eq. (9), $h_{0}n > \Sigma \lambda_{j} c_{j}$, i.e., the neutron consumption rate for the DNP production exceeds the delayed neutron generation rate. Therefore, by multiplying the parameter $h_{0}$ (or, which is the same, the parameter $\beta$) by some “efficiency factor”, we, contrary to traditional interpretations, will not increase but decrease the reproduction of the population. This fact unequivocally points to the fallacy of the usual interpretations of the “effective” parameter $\beta$.

### On the ratio of the fractions of prompt and delayed neutrons

An analysis of the nuclear reactor dynamics often begins with the statement “If the delayed neutron fraction is $\beta$, then the prompt neutron fraction is equal to $1 - \beta$.”
assuming it is applicable to any reactor states. On this basis, the expression for the rates of processes on prompt neutrons is presented in the following form (see, for example, (Sarkisov and Puchkov 1983, p. 312))

\[(1 - \beta)\\psi^p_m - \psi^m_m.\]  

(10)

From here, taking into account expressions (3) for \(\psi^p_m\), \(\psi^m_m\), the first term of Eq. (1) follows. However, even from expression (10) it can be seen that the parameter \(\beta\) is not related to the total fission neutron yield. The term \(\beta\psi^p_m\) is the consumption rate of prompt neutrons for the DNP generation, and, therefore, the parameter \(\beta\) expresses the DNP yield per one prompt neutron. In turn, the coefficient \((1 - \beta)\) has the meaning of the “effective” prompt neutron fraction, i.e., as the ratio of the rate of the positive contribution of prompt neutrons (to the total population of neutrons) to the total prompt neutron generation rate. Therefore, the statement cited above makes sense only applied to a single fission event, when \(\beta\) should be understood as \(\beta^p_m\) determined by relations (6). But as such, it has nothing to do with expression (10), which is written directly when compiling the velocity balance in Eq. (1).

The above thesis is specified, for example, as follows (Merzlikin 2001): “in the critical core at \(\Lambda = 0.001\) and \(\beta = 0.0064\) out of every hundred thousand fission neutrons, 99,360 are prompt, and the remaining 640 are delayed neutrons.” However, it follows directly from equations (1), (2) that this picture looks somewhat different.

In a population of \(n\) neutrons, \(n/\Lambda\) prompt neutrons, \((\beta/\Lambda)n\) DNP nuclei, and \(\Sigma\lambda/c\) delayed neutrons appear per second. Thus, in any state of the reactor at the specified values of the parameters \(\Lambda\) and \(\beta\), a thousand PNs and 6.4 nuclei of DNPs are generated every second per population neutron.

In the critical state, the generation rates of DNPs and DNs are equal: \((\beta/\Lambda)n = \Sigma\lambda/c\), so that \(\beta/\Lambda = 6.4\) delayed neutrons are generated per population neutron every second, i.e., 1006.4 neutrons are produced every second. In this case, the fraction of DNs in the total output \((\text{PN}+\text{DN})\) is obviously equal to 6.4/1006.4 = 0.00636 ≠ \(\beta\). Therefore, the wording of the above example should be changed as follows: “out of every 100,000 fission neutrons in the critical reactor, 99,364 are prompt and 636 are delayed ones.”

As we can see, in order to clarify the content of the EDNF concept, it is necessary to formulate a more accurate and detailed definition of the fractions of prompt and delayed neutrons (relative to both the total population and to each other). It should be kept in mind that models (1), (2) explicitly express only the rates of generation of PNs and DNPs, but not the rates of their loss. Therefore, it is impossible to determine the current number of PNs and DNPs and to establish their actual fraction in the population. It is possible to compare only the generation rate with the rates of other processes taken into account in the model, or with the current population size. The practical significance of this comparison lies in the fact that it makes it possible to estimate the contribution of the generation of PNs and DNPs at different reactivity values (Yuferov 2021).

According to equation (1), the parameter \(\beta = \beta^d_m\) is equal to the fraction of the generation of delayed neutrons relative to prompt ones (i.e., equal to the yield of DNs or DNPs per one prompt neutron):

\[\beta^d_m = \beta^m_m = \psi^d_m / \psi^m_m = \Sigma\lambda/c / (n/\Lambda).\]  

(11)

only in the reactor steady state, when the DNP generation rate \(\psi^d_m = h_p n\) and the DNP decay rate are equal, i.e., the delayed neutron generation rate. In this state, the parameter \(h_p\) expresses the population renewal rate, i.e., the relative delayed neutron generation rate is equal to \(h_p = \Sigma\lambda/c/n\). In the form \((\beta/\Lambda)n = \Sigma\lambda/c/n\), this relation means that in the reactor steady state, the prompt neutron consumption rate per DNP generation is equal to the delayed neutron generation rate. Of course, this equality should be regarded as formal, since models (1), (2) do not specify how and what neutrons of prompt or delayed origin are consumed for the DNP generation.

In unsteady states, the balance that determines the relative neutron population reproduction rate \(\alpha(t) = (dn/dt)/n(t)\), according to Eq. (1), is as follows

\[\alpha(t) = \rho(t)/\Lambda - h_p + \psi^d_m(t)/n(t) + Q(t)/n(t).\]

Representing here the relative delayed neutron generation rate as

\[\psi^d_m(t)/n(t) = \psi^d_m(t)/\left(\Lambda\psi^p_m(t)\right) = \psi^d_m(t)/\Lambda,\]

we will obtain the following expression for the delayed neutron generation fraction determined by relation (11) (i.e., \(\beta^d_m = \psi^d_m/\psi^m_m\)):

\[\beta^d_m(t) = (\beta - \rho(t)) + \Lambda(\alpha(t) - Q(t)/n(t)).\]  

(12)

For slow transients in the reactor operating conditions, when the terms \(\alpha\) and \(Q/n\) can be neglected, this expression is simplified as:

\[\beta^d_m(t) = \beta - \rho(t),\]

and shows that under the specified conditions \((\alpha = 0, Q = 0)\) in a supercritical reactor, the delayed neutron yield per one prompt neutron is always less than the similar DNP yield. At \(\rho = \beta\), relation (12) takes (at \(Q = 0\)) the form

\[\psi^d_m(t)/n(t) = \psi^d_m(t)/\Lambda = \alpha(t),\]

indicating the equality of the neutron population reproduction rate and the delayed neutron generation rate. The equality \(\rho = \beta\) also means that the neutron consumption rate per DNP generation \(\psi^d_m = (\beta/\Lambda)n\) is equal to the prompt neutron reproduction rate \(\psi^p_m = (\rho/\Lambda)n\). In this sense, it can be said that, at \(\rho = \beta\), prompt neutrons go completely to the DNP generation, and the positive population reproduction is due only to the contribution of delayed neutrons.
The given interpretations of the parameter \( \beta = \frac{\beta_{pn}}{\beta_{fn}} \), appearing in the reactor dynamics equations as the EDNF, and the relationship of this parameter with the delayed neutron fraction in the total yield \( \beta_{fn} \) are directly seen from the standard reactor kinetics equations, but, as we see, they differ significantly from the usual interpretations.

Some conclusions

A fairly obvious analysis of the reactor dynamics equations performed in this paper allows us, in particular, to state the following:

1. The parameter \( \beta \), which appears in the above equations as the “effective delayed neutron fraction”, is generally equal to the prompt neutron fraction consumed for the DNP generation, or the DNP yield per one prompt neutron in a single fission event.
2. Only in the reactor steady state, the delayed neutron yield per prompt neutron is equal to \( \beta \). At the same time, nothing can be said about the delayed neutron fraction relative to the total neutron population within the framework of models (1), (2).

3. In transient reactor processes, the delayed neutron generation fraction relative to prompt \( \mathbf{v}_{\mathbf{2}} / \mathbf{v}_{\mathbf{m}} \) is a variable value related to the parameter \( \beta \) by equation (12). This means that the thesis “if the delayed neutron fraction is equal to \( \beta \), then the prompt neutron fraction is equal to 1 – \( \beta \)”, often used in the analysis of reactor dynamics, is incorrect.
4. Multiplying the parameter \( \beta \) by the “efficiency factor” means, according to equation (1), an increase in the neutron consumption per DNP generation, i.e., contrary to traditional ideas, a decrease in the neutron reproduction in a supercritical reactor.

Thus, the usual definition of the parameter \( \beta \) as the “effective delayed neutron fraction” does not fully correspond to the physical content of this parameter and should be used only with appropriate reservations. In particular, it is necessary to distinguish between the fractions in single fission events, in reactor steady states and in transient processes. In the last two cases, we can talk about fractions of a relative total neutron population or about ratios of the process rates.

It seems that the proposed clarifications are appropriate both in terms of more adequate descriptions of processes in nuclear reactors and in relation to the formulations of nuclear safety requirements.

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