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Phenomenon of Cultural Dialogue in Development of Mathematical Culture of Students

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Abstract

The process of modernization of the present-day educational system, as well as the public demand for the formation of high school graduates' mathematical cultural skills are an important, but understudied, cause for concern.

The convergence of natural and mathematical studies and the Humanities through the concept of dialogue of cultures is becoming one of the high school's educational priorities. It enables graduates' continuous worldview formation and their readiness to practice self-directed learning. In this context, the purpose of this research is to provide theoretical justification and rationale for the formation of an educational environment which will enable a contextual approach to intercultural dialogue.

The article provides theoretical justification and clarification of the category of 'mathematical culture' and at submitting experimental proof of the need to create an educational intercultural environment.

The framework of the research includes methods of theoretical analysis, generalizations, scientific literature interpretation, as well as empirical research methods such as surveys and observation (search and standardized observation) and pedagogical experiment.

The paper presents a theoretical model of the levels of high school students' mathematical culture formation. The model's criteria and its measurements have been provided for. A set of didactic conditions for the development of high school students' mathematical culture based on the dialogue of cultures is identified.

The practice-oriented materials presented in the article will help schoolteachers in creating an intercultural educational environment that will enable the development of high school graduates' mathematical culture.

Keywords: educational environment, intercultural dialogue, mathematical culture.

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Introduction

The strategy of modernization of Russian education is determined by the global process of NBIC-convergence. Therefore, it becomes clear that a scientific trend is fundamentally changing the requirements for the organization of the educational process in the framework of basic education, particularly in its intermediate (middle) level. In this regard, education as a method of transferring experience to a young person turns into a mechanism for the development of his/her internalized culture, the process that determines the need to correlate the results of learning and the nature of the "culture" phenomenon.

Today scientists are paying particular attention to the idea of "cultural dialogue" as a possible foundation for the convergence of the Natural Sciences and Mathematics and the Humanities. Such convergence will contribute to the students' mathematical cultural development and their readiness for sustainable self-directed learning.

Despite the attention educators pay to the problem of each student's personal development, hardly all aspects have been investigated so far. This issue is of particular relevance since it is explained by the contradiction between two factors: the pedagogical community's awareness of the importance of the younger generation's higher mathematical cultural level on the one hand, and insufficient knowledge of the structural components of the category "mathematical culture" as viewed through the prism of the intercultural dialogue, on the other.

Dialogue of cultures between the Humanities and the basic mathematical education involves the creation of a specific educational environment that simulates intercultural contexts and enables high school students to understand the unity and universality of mathematics through the integration of Natural Sciences and the Humanities.

Purpose and objectives of the study

The purpose of the research is to provide a theoretical and experimental rationale for the development of educational intercultural environment which will ensure knowledge of mathematical culture among high school graduates.

- theoretical framework for studying the role of dialogue of cultures phenomenon in the development of the mathematical culture in a high school graduate is represented by:
 - culture-specific philosophical propositions (cultural approach) to constructing an educational intercultural environment (Bakhtin, 1986; Bibler, 1991; Lotman, 2000);

- research in the field of Science and the Humanities in teaching mathematics (Bashmakov, 2010);
- theoretical model of the “mathematical culture” category (Voronina & Moiseeva, 2012).

The propositions of the cultural approach allow us to consider the category of “dialogue” as a form and a means of cultural development, as well as a feature of consciousness (Bakhtin, 1986; Bibler, 1991; Lotman, 2000). Thus, Bakhtin (1986) speaks of “culture as a dialogue”. Bibler (1991) mentions "dialogue of cultures", and Lotman (2000) describes "dialogue in culture".

It is well known that the main message of any dialogue is the human need to communicate. Dialogue is the most important methodological principle of understanding and cultural self-development. Dialogue of cultures is a need for mutual understanding, which implies unity, similarity and identity (Bakhtin, 1986).

The point of view of Bibler (1991) on the dialogue of logics is of particular interest. The author believes that "by design, modern logic is the philosophical logic of culture, and by its form – the culture of logic" (Bibler, 1991, p. 5). Bibler interprets the dialogue of logics as "an expression of the universality of the intercultural dialogue" (Bibler, 1991, p. 5).

According to Lotman (2000), culture is immersed in a complex semiotic space (i.e. a space that represents a certain condition, a necessary prerequisite for communication), which is diverse and dynamic. That is the very space of cultural development.

The essential research concepts in the fields of Sciences and the Humanities as applied to the development of the dialogue of cultures environment need to be interpreted. Such interpretation in terms of mathematical cultural skills formation among high school graduates requires a full understanding of the essence of the ‘mathematical culture’ category.

As a result of our systematic and structural analysis of the essence of the category “culture”, we have found out that this category cannot be considered as some kind of invariant. It is important to consider such an important feature of culture as the connection of its objects with personal acceptance (or rejection), interpretation and reproduction (Kagan, 1996). That is why we are talking about the need to create an educational intercultural environment for the students to form the foundations of mathematical culture.

For further insight into the “mathematical culture” category it is necessary to talk about culture as a means of cognition and description of reality at various levels of abstraction using a mathematical language. The mentioned above confirms the idea that mathematics is the most important component of the general human culture and a characteristic of scientific, technical and social progress.

However, the concepts of mathematics and mathematical culture are not identical. The category “mathematical culture” will be used in our work to show the way the subject interacts with mathematical knowledge and to demonstrate the influence of mathematics on the structure and intellectual development of the subject’s personality (Maykova, 1996). Therefore, «mathematical culture” as an integrative personal construct will be discussed in our further analysis.

To define ‘mathematical culture’ category, we used a theoretical model proposed by Voronina and Moiseeva (2012). This choice of this method was determined by our understanding of the ‘mathematical culture’ category as an individual integral education.

According to the model proposed by Voronina and Moiseeva (2012), the category of “mathematical culture” is defined as a “personal integrative quality, which is the result of the interaction of several components: evaluative, cognitive, reflection and evaluation based and action-practice based components. These components are characterized by a value-based attitude to the obtained mathematical knowledge (evaluative component), by a high level of mathematical skills and knowledge (the cognitive component), by the ability to use the acquired mathematical knowledge and skills in practice (action-practice based component) and by the development of capabilities for further reflection of the process and the result of the mathematical activity (reflection and evaluation based component)” (Voronina & Moiseeva, 2012, p. 41).

Further discussion concerning the formation of high school graduates’ mathematical culture skills will deal with a systematic and purposeful process that involves the personal acquisition of mathematical culture that is required for one’s successful social adaptation to the processes of informatization and technology (Voronina & Novoselov, 2009).

Uspenskiy (2011) draws our attention to the process of integration of science and the Humanities in teaching mathematics, treating it as an urgent need for mathematics in rhetoric.

Studying different works dealing with the use of the sciences and the Humanities in teaching mathematics (Bashmakov, 2010; Kurant & Robbins, 2004) we have found out that they are in a subordinate relationship with the level of mathematical culture of a high school student. That is, the level of mathematical cultural development is determined by the interaction, mutual influence, and mutual enrichment of the rational scientific knowledge and irrational knowledge in the field of the Humanities. This gives us an insight into different ways of understanding the reality.

Thus, we came to the following conclusions:

- philosophical aspects of the cultural approach allow us to consider the category of “dialogue” as a form and a means of cultural development, as well as a feature of consciousness;
- mathematical culture is a complex integrative quality of a personality, distinguished by “evaluative”, “cognitive”, “action-practice based” and “reflection-evaluation based” components;
- the natural sciences and the Humanities are in a subordinate relationship to the “mathematical culture” of a high school student.

At this stage, the reasoning logic leads us to ask the following question: “What are the didactic conditions necessary for the development of a mathematical culture of high school students within the framework of dialogue of cultures?” We turned to an empirical study to find answers to this question.

Methodology

The methodological basis for the creation of an educational intercultural environment for the formation of a mathematical culture for a high school graduate features the cultural, pragmatic, and systemic approaches.

A cultural approach to building an intercultural environment involves immersion of a high school student into such an educational environment that makes it possible for a student to "move" from a lower level of formation of his mathematical cultural skills to a higher one.

Through the implementation of the pragmatic approach, culture acts as a projection of human activity as a focused activity of the subject. In view of this approach, the subject of the activity is considered by us from the perspective of the extensiveness of the mode “personal culture” indicative of each high school graduate. It is the pragmatic approach that directed us towards identifying certain didactic conditions for the development of the mathematical culture of high school students based on the dialogue of cultures phenomenon. Compliance with the pragmatic approach is ensured by the teacher’s indirect educational activities aimed at developing the mathematical culture of high school students. This is done by observing the whole complex of didactic conditions in the secondary school educational environment.

A systematic approach requires considering the category of mathematical culture as an integrated system, which includes a certain set of interconnected structural components. From the position of a systematic approach, the structural components of mathematical culture are “evaluative”, “cognitive”, “action-practice based” and “reflection-evaluation based” components.

The research is based on the methods of theoretical analysis, generalization, interpretation of scientific literature on the problem; empirical methods: questioning, observation (search and standardized) and pedagogical experiment.

The research was implemented in three stages:

At the first stage, a theoretical analysis of the existing methodological approaches in philosophical, psychological and pedagogical scientific literature was carried out; the research problem, its purpose, theoretical and empirical methods, the program of the pedagogical experiment were identified.

At the second stage, the criteria for the development of a mathematical culture of high school students were identified; a theoretical model of its high, middle and low levels was presented; a set of didactic conditions for the development of mathematical culture of high school students based on the dialogue of cultures phenomenon was revealed.

At the third stage, a pedagogical experiment was carried out to prove the effectiveness of the created intercultural educational environment, necessary to increase the level of high school graduates' mathematical cultural skills development.

To justify the theoretical and practical significance of the research and to identify practical difficulties associated with the organization of the activities that contribute to the development of school children's mathematical culture, we surveyed the students of the Institute of Education and Social Sciences of Pskov State University and teachers of mathematics of Pskov secondary schools. The survey showed that students' difficulties are associated with the determination of the components of mathematical culture and establishing new ways and methods of students' mathematical culture development. It should be noted that almost all of the respondents made emphasis on the following aspects as the components of mathematical culture: students' having computer skills and knowledge; knowledge of theoretical facts and of proving theorems; making neat notes. In fact, in practice, teachers quite often interpret mathematical culture as the ability of students to make calculations quickly, retell the text of a textbook, and solve a plot problem.

However, the development of these skills is associated only with the development of the cognitive and action-practical components of students' mathematical culture. In order to fully form a mathematical culture among schoolchildren, it is necessary to systematically develop all its components. In addition, we note that one of the most important problems stated in the concept of mathematical education is the problem of the content of mathematical education, appealing to the multifaceted and multidimensional understanding of mathematical education, in particular in the context of the dialogue of cultures.

As an instrument for developing the mathematical culture of high school students, we will consider the interactive cultural educational environment for teaching mathematics.

Basing on the analysis of the structure of the category “mathematical culture”, the requirements stated in the concept of mathematical education, the analysis of the practice of developing mathematical culture in a modern school, we have identified the following set of didactic conditions for creating a dialogue-cultural educational environment in the study of mathematical content.

1. Filling the educational process of studying mathematics with meaningful connections of mathematical objects and phenomena with facts and events of human culture. From the point of view of the concept of foundation, the influence of the humanities on the formation of the mathematical culture of students will be more intense if the process of mastering the corresponding mathematical content is interconnected at the level of dialogue of cultures, if there is an interaction and integration in teaching mathematical disciplines and disciplines of humanitarian and aesthetic and artistic blocks.

At present, mathematical education is one of the most important mechanisms for transmitting cultural values, norms, ideals, and meanings of life. The Federal State Educational Standard of Secondary General Education (2014) aims to study the mathematical content of “the formation of ideas about mathematics as part of universal culture, the formation of ideas of the significance of mathematics in the development of civilization and modern society; the formation of ideas about the social, cultural and historical factors in the formation of mathematical science; the development of ideas about mathematics as a universal language of science that allows us to describe and study real processes and phenomena, creating the conditions for acquiring initial experience in mathematical modeling” (Federal State Educational Standard of Secondary General Education, 2014).

2. Creation of educational problem situations involving the development in schoolchildren a holistic picture of the world in which mathematics is the language of science and culture; situations involving the translation into the language of mathematics of cultural phenomena, the formation of the ability to build a mathematical model of the whole variety of objects and phenomena of the world, to notice mathematical facts in objects far from mathematics. The perception and assimilation of new material in educational and cognitive activity is organized as a process of transferring information from one form to another. “Mathematics can be sensual. It has taste, it sounds and has color. It can be felt, and it can touch. It can describe the world. At the same time,

describing does not mean simplifying, reducing to diagrams and formulas, but presenting the world through the prism of mathematics, describing its problems and the probability of solving them by mathematical problems, transferring the world of one cultural language to another – the language of science” (Lipatnikova, 2015, p.156).

3. Educating the schoolchildren about the organization of cognitive dialogue using various languages. The dialogue within which the development of the mathematical culture of high school students is a special communicative medium that provides students with communication, reflection and self-development of the person, who is able to solve problems, generate new ideas, competently work with information. The cognitive component of mathematical culture presupposes the ability to analyze the meanings of the “humanitarian text of culture”, to extract and comprehend culturally significant information using mathematical methods; the ability to apply mathematical concepts in solving specific problems; the ability to discover mathematical concepts using examples of the sociocultural field.

The formation of students’ mathematical speech is facilitated by such types of work as the inclusion in the lesson structure of the dialogue forms of student interaction, discussions, and students’ independent work with practice-oriented cases on the issues being studied. The incentive for the active participation of students in communication will be differences in personal experience, students' worldviews, and assessments of problems discussed by students.

4. Practice proves that modern young people lack a focus on creative self-improvement; therefore, the creation of conditions for self-improvement of high school students is relevant. This implies the involvement of students in a creative environment, the creation of conditions for the development of social and value qualities of students. “No matter how artificially, and sometimes fabulously mathematical description is, it has its own morality. For the thinking scientist, the mathematical description has always been an inexhaustible source of surprise, born from the fact that nature shows such a high degree of compliance with mathematical formulas” (Arsenov, 2010).

To create a dialogue-cultural environment for the study of mathematical content in order to develop the mathematical culture of high school students, an analysis of the content of the basic school course in mathematics and disciplines of the subject field “Art” was carried out, meaningful lines of the study of mathematical material in the context of the dialogue of cultures were identified.

In order to implement the formulated provisions for the development of mathematical culture of high school students on the basis of the analysis of curricula in mathematics, the content and methodological component of the integrated course “Mathematics and Music” was developed:

- key topics are identified, the study of which can contribute to the development of value-appraisal, cognitive, action-practical and reflective-appraisal components of the mathematical culture of high school students, namely: “Mathematics of sound”, “Time as the measurement of music”, “Pythagorean scales”, “Mathematical laws of the tetrachord”, “The Golden Section in music”, “The geometry of music: geometric-musical transformations, harmonic symmetry”;
- theoretical material was selected; the form in which this content should be offered to students is defined, namely: computer presentation; cases for independent study of the topic; page on the site for independent work; video and audio recordings; creative workshops;
- practical-oriented and design tasks, quests, topics of small studies, situations for organizing discussions, etc. were selected for organizing group work.

The study of the discipline “Mathematics and Music”, in addition to the tasks of generalizing, systematizing and expanding the students' ideas about mathematical objects, expanding the understanding of the areas of application of mathematics, strengthening the applied, practical orientation of the content of mathematical training of graduates, expanding the general cultural horizons of students by introducing them the best examples of art, has the goal to develop the mathematical culture of students.

The substantive component of the course “Mathematics and Music” were the cases presented by a set of competency-based teaching materials for students to work independently, as well as questions and tasks on the textual content of the case.

Results

The purpose of this empirical study was to demonstrate the possibility and expediency of organizing the developed dialogue-cultural environment, aimed at developing the components of the mathematical culture of high school students in the process of studying mathematical concepts and techniques of mathematical activity in high school.

The base of the study of the formation process of the mathematical culture of high school students was Municipal Budget Educational Institution “Secondary School No. 18” in Pskov, students of the 10th “A” class.

We assessed the motivational-value component of mathematical culture with the following indicators: the student's value orientations, his or her motivation to study mathematical disciplines, and the level of "creativity" when performing various types of projects. To assess the level of formation of this component, the following methods were used: questionnaires, observation, analysis of students' creative projects. Creative projects of students were evaluated according to the following indicators: the content of the project, reflecting the student's ability to work with literature, to make conclusions; design of the project, indicating the ability to structure the material logically; independence of the student when working on the project.

The cognitive component of mathematical culture is associated with the mastery of mathematical concepts and mathematical skills by students. Mastery of mathematical concepts was assessed by indicators of completeness and reliability; methodology by Usova (1980).

The effective-practical component (the ability to use the obtained mathematical knowledge and skills in practical activities) was assessed with the indicators of completeness, reliability, awareness; methodology by Usova (1980).

The reflective-evaluative component of mathematical culture was identified by such indicators as attitude to oneself and the results of one's educational and cognitive activity.

In the framework of this experimental study, we created a model of three possible levels of formation of the mathematical culture of high school students: high, intermediate, low.

The level of characteristics presented below reflects the consistent development of mathematical culture among secondary school students from a less than perfect level of formation of the components.

At a low level, the student is not motivated to study the disciplines of mathematical content. The student is capable of reproductive memorization, reproduction of knowledge. However, knowledge about the possibilities of the practical application of mathematical concepts and methods for solving applied problems is not systematized. There is no ability to translate the information of the cultural field into a signed form; a low level of formation of mathematical speech is noted. The desire for self-actualization at this level was not found. The student has low self-esteem in the field of mathematical preparation (he is not able to evaluate his own motives, goals). This level is characterized by a lack of independence in the performance of cognitive activities.

At an intermediate level, the student has an interest in mastering mathematical knowledge, the need for self-education, the development of skills in applied mathematical content. The student demonstrates a sufficient level of knowledge in the field of mathematics, is partially aware of the importance of mathematical knowledge in solving applied problems, and is able to translate cultural field information into a symbolic form. In accordance with the intermediate level, individual skills and abilities to apply mathematical methods and techniques for analyzing practical situations of mathematical content have been formed. The student has an interest in historical information in the field of culture and art, in the studied processes of the surrounding reality, needs information that would increase his cultural level. A secondary school student is diagnosed with a desire for self-actualization. A schoolboy occasionally evaluates his own motives, goals, and reflects in cognitive activity, analyses of gaps in knowledge and skills.

The high level is characterized by the formation of cognitive motives among high school students, and the demonstration by students of sufficient knowledge in the system of mathematical concepts. At this level, there is a desire and ability to adjust their own mathematical activity, to determine the goals and means of personal mathematical development. The student has a high level of skills in applying methods and techniques of mathematical activity in the analysis of the phenomena and events of the surrounding reality. The student feels the need to obtain new mathematical knowledge, skills and abilities, realizing their importance for self-realization. As a result, the student has developed advanced mathematical speech, allowing you to transcode information into various mathematical languages. The student has a strong desire for self-actualization, he knows how to critically evaluate his own motives, goals of cognitive activity has a high level of independence in cognitive activity.

Testing the developed dialogue-cultural environment when teaching senior pupils' mathematics at the stage of a control stage showed that the implementation of the formulated didactic conditions in the educational process contributed to an increase in the level of formation of the mathematical culture of senior pupils since there was a positive dynamics in the increase in the number of students with a high level of development of mathematical culture compared to the initial data: the number of students with a low level of mathematical culture decreased by 18.6%, the number of students with a high level of development of mathematical culture increased by 26.1%.

The number of high school students at a high level of development of the mathematical culture increased by 4.5 times, while the average level of development of mathematical culture of students remained virtually unchanged. The number of high school students with a low level of mathematical culture, on the contrary, decreased by 3.5 times. It should be noted that the motivation of students in the study of mathematics has increased significantly.

A quantitative assessment of the results of the pedagogical experiment made it possible to identify the effectiveness of the proposed set of didactic conditions aimed at increasing the level of development of the mathematical culture of high school students. In general, it should be said that the process of the formation of the mathematical culture of high school students in the implementation of the interactive cultural environment for teaching mathematics in high school is productive.

Conclusion

The solution to the problem of the formation of mathematical culture among high school students will be facilitated by the appeal in the process of studying mathematics to the phenomenon of dialogue of cultures.

According to the purpose of the study, a set of didactic conditions for the development of the mathematical culture of high school students based on the phenomenon of dialogue of cultures was identified, the content and methodological component of the integrated course “Mathematics and Music” were highlighted. The experiment showed the effectiveness of the proposed set of didactic conditions aimed at increasing the level of development of the mathematical culture of high school students.

Basing on the results of the study, we can formulate the following conclusions. First, currently, there is a need to reconsider the goals, objectives, content, and technologies of the school course of mathematics, taking into account the changes taking place in modern society. In the modern educational space, the formation of the mathematical culture of schoolchildren is becoming one of the most important tasks of both mathematical and general cultural education of schoolchildren.

Second, cultural, subject-activity, and systematic approaches to the analysis of the problem of the formation of the mathematical culture of students lead to the need to create an educational dialogue-cultural environment for the formation of the mathematical culture in a high school graduate while systematizing the school course in mathematics.

Third, the set of didactic conditions for the formation of the mathematical culture of schoolchildren proposed in this study, the subject and methodological content of the course “Mathematics and Music” developed on its basis, contributes to the formation of the mathematical culture of a high school graduate.

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