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Teacher Challenges in Articulating Not Knowing While Solving Geometric Reasoning Tasks

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Abstract

Not-knowing is an underexplored concept defined by an individual's ability to be aware of what they do not know as a means to plan and more effectively face complex situations (Mason & Spence, 1999). This qualitative study focuses on analyzing students' (n=10) ability to express their "not-knowing" while completing geometric reasoning tasks and reflecting periodically. It becomes evident that the students have difficulty expressing their not-knowing. Through transcription analysis, meaning coding, and interviews, four recurring themes emerge. Each theme will be discussed followed by a conclusion of their overall importance in relation to students' ability to express not-knowing.

Keywords: Teacher not-knowing, secondary school mathematics, geometric reasoning.

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Introduction.

Not-knowing is the first step to understanding, carrying an important value in learning. Mason (1999) claims, "awareness of knowing and of not knowing is crucial to successful mathematical thinking." Tahta (1972) uses not-knowing as a means to describe an algebraic process of finding what we do not know with the use of what we do know. Shah (1968) quotes ancient wisdom according to which not-knowing is a critical state because from it knowing can follow. However, little research has been done to understand the not-knowing phenomenon. In this study, we are examining the following *research questions*: how do students express what they do not know while solving geometric reasoning tasks? And, what challenges do they face in externalizing the not-knowing?

Theoretical Framework.

Even though the topic of not knowing has not been extensively researched, we believe that the concept is related to the ideas of noticing detailed in *Mathematics teacher noticing* (2011). In this book, Mason (2011) discusses teacher noticing of students' mathematical thinking. While the focus itself is on teachers noticing student thinking, he also describes the educational benefit of noticing, stating that it is used to "alert oneself in the future so as to act freshly rather than automatically out of habit" (p. 35). It is this concept of noticing one's own thinking or to "alert oneself" that resembles our ideas regarding not-knowing.

Examining teachers' ability to first notice then decide how to respond accordingly, mirroring not-knowing, Jacobs, Lamb, Philipp, and Schappelle (2011) argue that an individual must first become aware of what they do not know to plan and respond effectively to the given situation. Not knowing is the first step to learning, and if an individual becomes aware of what s/he does not know and use it to their advantage, it is possible for them to become more efficient and effective learners.

Being aware of one's own not-knowing is a spark that could potentially ignite inert knowledge. Renkl, Mandl, and Gruber (1996) suggest that "the problem of inert knowledge is surely of major educational importance". The metacognitive process involving awareness of not-knowing allows an individual to take a step back and analyze the situation to plan accordingly. This gives an opportunity for inert knowledge to come forth that would otherwise not have been used when acting out of habit. Emphasizing the role of problem solving, Carlson and Bloom (2005) claim that a good problem solver exhibits flexibility as well as powerful math related processes to arrive at their solution. They also state that those who solve the problems do not solely rely on heuristics. The awareness of not-knowing opens the doors to becoming more flexible at solving problems while becoming less dependent on heuristics. Furthermore, Simon (1996) details "transformational knowledge" as a way to assess a given situation and select the best possible outcome. This type of knowledge could potentially be achieved if an individual becomes aware of her not-knowing and use it as a means to understand a given situation.

While the importance of not knowing as a first step to knowing may be clear, the ability to be aware of the not knowing in the moment may not be an easy task. Mason (1999) details the difficulty of knowing-to-act in the moment, and we believe that being aware of your own not knowing in the moment may be just as difficult. With that thought in mind, we aim at understanding how students are able to express what they don't know as a means to identify a possible connection to their learning.

We will be examining students' not knowing in the context of problem-solving. Mason and Spence (1999) observed the situation when "most people have been stuck on a problem, and vaguely aware of being stuck" (p. 147). They further explain the situation of "being stuck", stating: "…as long as their awareness of their state remains below the surface of action they are unable to act upon that awareness by intentionally invoking some strategy" (Mason & Spence, 1999, p.147). However, the awareness of the state

is not efficient enough to "unstuck" the situation if it is not supported by prior knowledge. "Strategies come most readily to mind if they are richly linked with past experience" (Mason & Spence, 1999, p. 147). In this sense, someone's awareness of not knowing is informed by his/her prior knowledge. Prior knowledge serves as a base of someone's knowing and not knowing. Therefore, we use connections between prior knowledge, knowing, and not-knowing as a guiding frame in examining the not knowing phenomenon (figure 1).

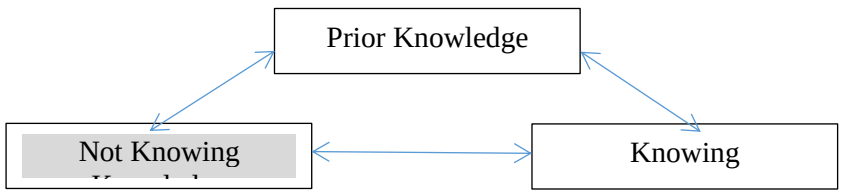

Figure 1: The frame for examining students' not knowing

Methodology.

This qualitative study focused on students' articulation of not-knowing and the challenges they faced during this process. Ten students were selected for the study at a university in the southwestern border region of the United States. These students were enrolled in a teacher education course on Geometric Reasoning, which focused on problem-solving using a tangram (a seven-piece puzzle) to construct squares with a different number of pieces. This setting was used to collect data and analyze student externalization of their not-knowing. Data sources consisted of audio recording, reflections, and interviews described below. We believe these methods will provide data that will directly reflect student abilities at expressing their not knowing. Through audio recordings, written reflections, and interviews, we aim at capturing student expression of not-knowing in more than one way.

Procedure.

During the class, students worked on various geometric reasoning tasks using the tangram puzzle. The tangram consists of seven 2D geometric shapes: two large isosceles right triangles, one medium size right isosceles triangle, two small isosceles right triangles, one square, and one parallelogram. Before engaging in problem-solving, the students were introduced to the tangram by learning about the tangram shapes and their properties. It helped students to activate their prior knowledge. Then, students were solving tasks by constructing squares with a different number of the tangram pieces. The most challenging tasks – constructing a square with 5, 6, and 7 pieces in standard (a problem has a solution) and non-standard (a problem does not have a solution) situations were audio recorded.

Recordings and Transcriptions.

At four different points during the course, students paired up and audio-recorded each other while attempting to create a given square using the tangram pieces. The task was to complete a square using a given number of pieces while vocally expressing what they knew and did not know at that specific moment. First, the participants recorded the seven-piece attempt, followed by a second attempt at the seven-piece, then the six-piece, and finally the five-piece.

The audio recording of the participants while attempting the task meant to bring forth their thought process of what they knew and didn't know at the time. This was the reasoning behind the audio recordings, as it was thought to be a way of encapsulating students' not-knowing as a data source for analysis.

Reflections.

Two major written reflections were assigned during the course containing three to five questions respectively. The participants were asked to reflect through writing on certain ideas discussed as well as their knowing and not-knowing. These reflections asked the students to express what they knew and didn't know during the time of the task. Their answers could be used to analyze how aware they were and their ability to express their not knowing reflectively.

Interviews.

Two students out of ten were selected for semi-structured interviews conducted at the end of the course to analyze student reflective thoughts on their learning. Students were purposefully selected based on their successful learning in the course. The interview consisted of thirteen questions focused on extracting student not-knowing in a reflective fashion, which was also audio recorded. The audio recordings were then transcribed and analyzed.

Data analysis.

In order to analyze the transcriptions, reflections, and interviews, meaning coding, meaning condensation and interpretation techniques were used (Kvale & Brinkmann, 2009) as the main methods of analysis. The first set of analysis consisted of transcriptions from the participants' attempts at creating the seven, six, and five-piece squares using the tangram while voicing their knowing and not knowing. There were two methods of analysis used for this part. First, the transcriptions of each individual participant were separated and analyzed to examine how well the participant expressed their not knowing at different points in time during the course. Second, an analysis of the transcriptions as a whole was conducted, in an effort to encompass key similarities between them. The second set of analysis was two reflections in which students reflected on what they knew and did not know at the given time based on the course. Meaning coding technique was used to interpret their ideas and make connections to their transcriptions. Finally, the last part of the analysis included the two interviews. The interviews are meant to provide a closer look into the thoughts of the two participants. The questions are aimed at evoking not-knowing reflectively and active not-knowing while the interview is being conducted. Meaning interpretation and meaning condensation were the primary tools in analyzing this data to make connections between other forms of not-knowing expressed throughout this study.

Findings.

The data analysis clearly demonstrated that the participants had difficulties expressing their not-knowing. The analysis shows that there are several recurring statements made by the participants. These statements were categorized and the following four major themes (deflection, pressure, lack of heuristic sense, and fractured prior knowledge) emerged that will be described below.

Deflection.

Deflection of not-knowing can be identified as avoidance of challenge when an individual shifts the focus of their not-knowing somewhere else besides themselves. Below is an example of the participant's response, which demonstrates deflection of not-knowing:

> Student: I know as far as formulas and all that they are not going to help me at all. It's more of a pattern thing and if students tried to do this it would be the same thing for them…

The participant makes it a case that she believes formulas will not help her at arriving at a solution, inferring that she does not know how to arrive at the solution. Her thought of not-knowing takes a shift stating that if students tried, it would be the same for them. Why is it that when asked about her own not-

knowing she deflected? Instead of being aware of her not-knowing as a first step to finding the answer, the individual deflects to what she believes others don't know. A total of five out of the ten participants deflected at one point or another throughout the analysis.

Pressure.

Participants expressed pressure through direct vocalization, frustration, or sense of urgency. Every one of the participants demonstrated pressure at different points during the transcription and reflection sections. Let's take a look at these two statements below:

> Student 1: Makes no sense to me. Jesus Christ… Ok, so it doesn't make sense... Jeez. It's almost something. Oh, God. Can I make a rectangle?
>
> Student 2: This is ridiculous (laughs). Putting these squares together. It's a lot harder than I thought it would be. It's destroying my idea of what a square is… Maybe if I … no. Oh my God, this is so much harder than I thought it would be. I think you did that. Then we can put a little triangle here. I think that's what you had, isn't it? No. Argh, this is so frustrating.

These two students demonstrate pressure, which may be a factor impeding awareness of not-knowing. The first student demonstrated clear frustration throughout her thought process at his inability to make sense of the situation. The second student directly expressed her frustration, derived from expecting the task to have been easier than expected.

Lack of heuristic sense/trial-and-error.

Every single participant attempted the first task through trial and error as demonstrated by students' transcriptions. We hypothesize that in seeking the solution; students may become "tunnel visioned" in the process of trial and error, clouding their awareness of not-knowing. In that regard, it affects the individual's ability to invoke a sort of strategy to overcome the problem. Below is part of a transcription, where a participant demonstrated "tunnel vision."

> Student: I am going to start with the parallelogram just because it has the oddest shape. And ... I'm going to try to make the sides even, and I'm just trying to add from there but it doesn't fit. So I don't know how to get the sides to be straight without having any leftovers. Ummm ... ok. No, I'm going to start again. I'm going to start with the big pieces now. I'm going to put the two triangles together. Okay, I'm going to put the two big triangles and try to make everything fit in the middle. Okay, so I'm putting some and they don't fit but I'm kind of getting the shape, kind of not.

The attempt demonstrated above gives a glace at a student's thoughts as they attempted to solve the tangram. Trial-and-error is known to be the most common heuristic as it is simple to use and eases the cognitive load on the individual. Unfortunately, sticking to trial-and-error was ineffective, as the student was unable to solve the tangram. Most of the participants, even with prior knowledge, stuck to trial and error to the very end. Perhaps, inability to evoke awareness of their not-knowing was a factor that led to not solving the tangram.

Fractured prior knowledge.

Whereas lack of heuristic sense deals with strategy, fractured knowledge is present when a participant may have prior knowledge gaps within the given topic, have misunderstandings of the topic, or simply lack the prior knowledge required for the given topic. We argue that if an individual has fractured knowledge, it will directly impede their ability to invoke not-knowing as a means to gather knowledge that simply is not there or is "fractured." Below is a representation of a participant's response with fractured prior knowledge:

> Student: Ok, so that might be too long. So, I think it has to be smaller than 2 and square

root 2. Maybe it can be 3? I will try for 3.

While attempting to find the side length of a square that must be constructed, the student makes the revealing statement above. There is a clear misunderstanding of the number $2\sqrt{2}$ ("two square root of 2") in comparison to the number 3. The student believes that 3 is smaller than $2\sqrt{2}$ and carries on without a second thought, guiding her down the wrong path. In this sense, even if awareness of not-knowing were to occur, fractured knowledge will be the deciding factor in leading the student farther away from the answer. For this reason, we believe there is a close connection linking prior knowledge to both knowing and not-knowing (figure 1). In this example, we are particularly interested in the connection between prior knowledge and not-knowing, as the student's fractured prior knowledge is a contributing factor to both their inability to invoke not-knowing awareness as well as leading them astray.

These four themes frequently emerged throughout participant transcriptions, reflections, and interviews. Even though some of these themes emerged less frequently than others, they all hold importance, as they reflected challenges in participants' ability to express not-knowing.

Connections to learning.

The data gathered from the students clearly demonstrated the difficulty faced when reasoning with the tangram activity. The audio recordings displayed growth in understanding from all students, some better than others. Once the lesson was over, two students stood out who displayed almost complete mastery of the tangram. While all the first audio recordings displayed difficulty in student ability to express their not knowing in the moment, these two students demonstrated to be very active thinkers as they attempted the task. Even though they also faced difficulty clearly expressing not knowing, they demonstrated more effective attempts as opposed to counterparts seemed to have contributed to their learning. These two students were interviewed as a means to better understand the progress made. Their interviews gave insight into their thinking during the attempt to construct the tangram. Both students expressed their "hate" for having to express what they didn't know in the moment. One student stated that the question of "what don't you know?" seemed like a really simple question, but in fact found it to be really hard to answer. It appeared that these students disliked the question because it was difficult to articulate what they didn't know at the moment. The important factor is that these students made the attempts to express their not knowing. Let's take a look at these two excerpts below:

Student 1: So I have this big triangle in the corner and I keep putting it there because it seems like it makes sense there but I don't know if this is the right way to go I wonder if there is more than one way to do it.

Student 2: It can't be two square root of two, I tried that already. So it can't be two square root of two because it's the largest we can have which will give me an area of eight. So it has to be less than two square root of two and I tried side length of 2, but it limits the amount of pieces I can use. I am wondering if this is even possible.

Student 1 attempt may have been flawed, but her ability to step out of the problem and question if there are different possible ways to go about it is a line of thinking that may have led her to her correct answers later on. Student 2 struggles with constructing a square with 6 pieces, however, her not knowing leads to think about what she does know, arriving at her conclusion.

These two students responses and performance contrast directly with other participants who made little to no attempts at expressing their not knowing or simply lacked the ability to do so. Those who made little attempts of expressing not knowing throughout the lessons seemed to have struggled more extensively. While both parties attained new knowledge from the course, the results indicate that those students who more actively sought to express their not knowing gained a better understanding of the concept.

Discussion and Conclusion

It is important to note that since the topic of not-knowing is fairly new, there is no clear example of what perfect expression of not-knowing looks like. This presented itself as an obstacle when we analyzed the data since there was nothing to compare to. Even then, the data and its analysis demonstrated that some students could express their not-knowing better than others, leading them to demonstrate stronger conceptual understanding by the end of the course. For instance, one of the students in her reflection after the Tangram task with 6 pieces indicated "I would still like to know/explore what other figures would not be possible to create because of restricted areas" expanding her awareness of knowing/not knowing to solve similar or extended problems in the future. In this regard, the results of the study contribute to advancing the overall conceptualization of the construct of the not-knowing and its relationship with prior knowledge and knowing (figure 1).

Students "deflecting" their not-knowing should not come as a surprise, as not many individuals are fond of admitting their lack of knowledge or understanding. Nevertheless, the presence of deflection in the analysis shows how someone may shift their not-knowing onto someone or something else, rather than accepting their not-knowing and use it as a means to find a solution to a given problem.

The theme of "pressure" did not come as a surprise either. It may be closely related to Krashen's affective filter hypothesis (1985), which details students' abilities to learn based on what other thoughts might be on their mind. In this study, we saw pressures caused by time, frustration, and even fear of being judged by others.

The "heuristic sense" theme observed in the study is one deeply rooted in students' minds. Since one of the most basic heuristics is trial-and-error it is easily available for anyone to use. Trial-and-error was the guiding force in decision making for many participants from the start of the course, and in some cases, to the end of the course. Students may be diverting to trial-and-error heuristic to ease the cognitive load.

Not-knowing can serve as a spark in creating a plan to solve a problem, but if fractured knowledge exists, it is likely that awareness of not-knowing may yield an incorrect answer. Just like in the example concerning fractured knowledge, it can be determined that as long as the individual has fractured prior knowledge, it will impede awareness of not-knowing. Therefore, prior knowledge itself should be connected in order to assist students' in expressing their not knowing.

Mason and Spence (1999) make it clear how important not-knowing is as a first step for knowing to occur. This study focused on finding out how students are expressing their not-knowing. Results demonstrated that the participants had difficulty expressing their not-knowing while solving geometric reasoning tasks.

Four recurring themes came to light from analyzing participants' transcriptions, reflections, and interviews. Examples of deflection, pressure, lack of heuristic sense, and fractured knowledge were discussed to demonstrate how each affected not-knowing awareness. Understanding these themes can better assist in minimizing the problem while maximizing the potential of not-knowing as a lead to knowledge and understanding. Students that were better able to express there not knowing ultimately demonstrated a stronger understanding of the concept at the end of the course. That being said, it is still evident that all students had difficulty expressing their not knowing. These findings might serve as a stepping-stone to further research not-knowing, as it may directly link to more effective and efficient student learning.

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