

VII International Forum on Teacher Education

Future math teachers` readiness to develop mathematical abilities of pupils

Nailya V. Timerbaeva* (a), Elmira I. Fazleeva (b), Kadriya B. Shakirova (c)

(a), (b), (c) *Kazan Federal University, 420008, Kazan (Russia), 35 Kremlyovskaya street, timnell@yandex.ru, elmira.fazleeva@mail.ru, shakirova_ka@mail.ru*

Abstract

Current study is devoted to the readiness of future math teachers to develop pupils` mathematical abilities. Development of pupils` mathematical abilities is one of the most important problems in learning mathematics. Many Russian and foreign scholars were engaged in research of mathematical abilities.

Tutoring support having been provided for young teachers and graduates of Kazan federal university pedagogical department showed a number of issues. Novice teachers engage "average" students, do not implement an individual approach, have certain difficulties in determining pupils` mathematical abilities and applying the methods of its development. Consequently, the purpose of this study was to identify ways to improve training of future mathematics teachers in terms of developing students' mathematical abilities.

Analysis of specific literature showed distinction between ordinary "school" abilities to assimilate mathematical knowledge, to reproduce and implement it on the one side, and creative mathematical abilities associated with designing original product. In this regard questionnaires were drawn to establish ideas of KFU students as future math teachers about current state and possibilities of developing pupils` mathematical abilities.

The study determined that success of future teachers` training depends on many factors. A considerable place must be given both to development of students` mathematical abilities themselves and to development of their pedagogical competencies.

Keywords: future teachers` training, pupils` mathematical abilities, mathematical abilities development.

© 2021 Nailya V. Timerbaeva, Elmira I. Fazleeva, Kadriya B. Shakirova

This is an open access article distributed under the terms of the Creative Commons Attribution License (CC BY 4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Published by Kazan federal university and peer-reviewed under responsibility of IFTE-2021 (VII International Forum on Teacher Education)

* Corresponding author. E-mail: timnell@yandex.ru

Introduction

In modern realities, the most important task of teaching mathematics is development of students' abilities. Scientists such as Gusev V.A. (2003), Krutetsky V.A. (1968), Leites N.S. (2003), Teplov B.M. (1961), Yakimanskaya I.S. (2004) and others were engaged in the study of mathematical abilities. These abilities are closely related to learnability and intellectual properties that determine it as generalization, awareness, flexibility, stability, and independent thinking. For mathematical abilities development it is necessary to know the ways of its identifying and methods of its formation, to understand their structure and essence. (Matyushkin, 1991).

Tutoring support has been carried out for young teachers – graduates of Kazan federal university since 2015 (Shakirova, Fazleeva, Timerbaeva, 2020). Observations showed that beginning teachers had certain difficulties in performance related to: the content of Math discipline (insufficient level of generalization and systematization of mathematical knowledge, training of techniques, methods of action, algorithms), with means and methods of pedagogical influence on students (difficulties in setting and solving pedagogical problems, inability to take into account past mistakes, insufficient flexibility in modifying tasks during the lesson, inability to organize educational and cognitive activities of pupils); with individual psychological characteristics of the teacher (temperament, volitional qualities, emotional sphere, etc.); with insufficient reflection and low self-criticism. Young teachers do not consider the reasons for their failure, disadvantages of their own work, they are concentrated on a 'average' pupil, do not implement individual approach, they find it difficult to determine pupils' mathematical abilities and apply methods of its forming. A novice teacher is still unable to provide high-quality mastering of mathematical concepts, algorithms, properties and methods of action.

Purpose and objectives of the study

The primary purpose of this study was to determine the ways to improve future math teachers' training for developing pupils' mathematical abilities.

Literature review

A large number of Russian and foreign researchers were engaged in the study and development of mathematical abilities. We understand mathematical abilities as individual and psychological characteristics of human activity in studying and creative developing of mathematics.

Consequently, in the process of teaching schoolchildren it is necessary to develop natural inclinations of students (inclinations are innate, genetically determined features of the central nervous system or individual analyzers, which are prerequisites for development of abilities), bringing them to abilities that are already formed in performance (Druzhinin, 1999).

Morduhay-Boltovskiy (1988) in his works highlighted the following components of mathematical abilities:

- "strong memory", memory for "objects of the type which mathematics deals with", memory rather not for facts, but for ideas and thoughts;
- "wit", which is understood as ability to "embrace in one judgment" concepts from two loosely connected areas of thought, to find in what is already known and that is similar to given, to find similar in the most distant, seemingly, completely dissimilar objects;
- quickness of thought (speed of thought is explained by the work that unconscious thinking does to help the conscious one). Unconscious thinking, according to the author, proceeds much faster than conscious one.

A considerable number of works on this issue were published abroad. The possibility of developing mathematical abilities of a large number of students by solving non-standard and logical problems, using creative approaches to teaching was considered in works by Silver (1997). The study of Sriraman, Haavold and Lee (2013) is devoted to the study and differentiation of such concepts as "mathematical creativity", "mathematical giftedness", "mathematical ability". The work of Jonsson, Norkvist et al. (2014) is devoted to the choice of methods that contribute to development of creative mathematical reasoning of students. The use of dynamic mathematical programs in solving mathematical problems as an opportunity for the development of creative thinking of students was studied in works of Granberg and Olsson (2015).

Famous Soviet mathematician, world-renowned scientist, academician Kolmogorov (2001) distinguished the following basic mathematical abilities: 1) ability to transform skillfully complex letter expressions that do not fit standard rules, or, as mathematicians call it, computational (algorithmic) abilities; 2) geometric imagination, or geometric intuition; 3) art of consistent, correctly divided logical reasoning. He believed that different aspects of mathematical ability can occur in different combinations, usually appear quite early and require continuous exercise.

We agree with Soviet psychologist, specialist in the field of educational and developmental psychology Krutetkiy (1968) who reveals the essence of mathematical abilities as follows:

1. Ability to generalize mathematical objects, relationships, and actions.

2. Ability to curtail the process of mathematical reasoning and the system of corresponding actions.
3. Flexibility of thinking processes (free switching from one mental operation to another).
4. Striving for clarity, simplicity and cost-effectiveness of solutions of solution.
5. Reversibility of thinking process in mathematical reasoning (ability to switch quickly and freely from direct to reverse thought).

Thus, one should distinguish between ordinary "school" abilities for mastering mathematical knowledge, for their reproduction and independent application, and creative mathematical abilities associated with the independent creation of an original product. The article is focused on the "school" ability to master mathematical knowledge.

Methodology

The research study employed two types of methods: theoretical (analysis of pedagogical, psychological, scientific and methodological literature); and empirical (conversation, observation, questionnaire).

Based on analysis of specific literature the questionnaire was compiled to identify the presence of mathematical abilities in students and disclose their ideas about the possibility of developing such abilities in pupils. Questionnaire consists of mathematical and methodological parts. Research involved 148 participants. The study restricted its coverage to KFU 2nd and 4th year Bachelor of Pedagogical Department students and 1st year Master of Nikolai Lobachevsky Institute of Mathematics and Mechanics students, and also 1st year Bachelor of Pedagogical Department students from Leo Tolstoy Institute of Philology and Intercultural Communication.

Mathematical part of questionnaire involves following tasks available in 2 variants.

Table 1. Mathematical part of questionnaire.

Variant 1	Variant 2
1. Solve equation: $x^2 - 4x - 21 = 0$.	1. Solve equation: $x^2 + 2x - 15 = 0$.
2. Solve inequalities:	2. Solve inequalities:
a) $4x^2 - 4x + 1 > 0$, б) $4x^2 - 4x + 1 \geq 0$,	a) $9x^2 - 12x + 4 > 0$, б) $9x^2 - 12x + 4 \geq 0$,
в) $4x^2 - 4x + 1 \leq 0$, г) $4x^2 - 4x + 1 < 0$.	в) $9x^2 - 12x + 4 \leq 0$, г) $9x^2 - 12x + 4 < 0$.
3. Perform an action: $12\frac{1}{3} - \frac{5}{7}$.	3. Perform an action: $13\frac{1}{7} - \frac{4}{9}$.
4. Convert to degree measure: $\frac{11\pi}{12}$.	4. Convert to degree measure: $\frac{7\pi}{15}$.

Offered tasks in this part are aimed at identifying the presence of such mathematical abilities as ability to curtail the process of mathematical reasoning, flexibility of thinking processes, striving for clarity, simplicity and cost-effectiveness of solutions, ability to generalize mathematical actions.

It is necessary to decipher the essence of each of them.

Ability to curtail the process of mathematical reasoning:

- do not write out the formula in detail when calculating the discriminant of a quadratic equation with a negative intercept, for example, find immediately $D = 16 + 84$, а не $D = 16 - 4 \cdot (-21)$;

- omit some mathematical actions, for example, convert mixed fractions to incorrect ones, bypassing the rule of decreasing by one integer part of the reduced fraction, in this case, $12\frac{1}{3} - \frac{5}{7} = \frac{37}{3} - \frac{5}{7}$, but not as in the following case $12\frac{1}{3} - \frac{5}{7} = 11\frac{4}{3} - \frac{5}{7}$.

Flexibility of thinking processes:

- use Viet's theorem when finding the roots of a quadratic equation or apply the discriminant formula for an even second coefficient $D_1 = 4 + 21$;

- pre-reduce fractions when calculating product, for example, $\frac{7\pi}{15} = \frac{7 \cdot 180^\circ}{15} = 7 \cdot 12^\circ$, а не $\frac{7\pi}{15} = \frac{7 \cdot 180^\circ}{15} = \frac{1260^\circ}{15}$.

Striving for clarity, simplicity and cost-effectiveness of solutions:

- select a complete square immediately without calculating the discriminant $4x^2 - 4x + 1 = (2x - 1)^2$.

Ability to generalize mathematical actions:

- when solving inequalities from the second task, use the same drawing, etc.

Methodological part was offered to 3rd, 4th and 5th year Bachelor and Master students. It consisted of 4 questions.

Methodological part

1. What criteria can be used to assess the presence of mathematical abilities?
2. How can you develop your math abilities?
3. How are mathematical abilities expressed?
4. How did your teacher develop your math abilities?

Results

Here are the results of the mathematical part of the tasks.

Table 2. Results of the mathematical part of the tasks.

Components of mathematical abilities	Years, number of participants, %					Master students
	1	2	3	4	5	
Use of Viet's theorem	13	32	30	29	6	40
Use of discriminant formula for an even second coefficient	6	0	15	7	18	60
Concise notation of discriminant formula	88	88	77	71	64	60
Selection of complete square without calculating discriminant	13	28	46	43	36	60
Solving inequalities	88	88	70	90	82	100
Rational actions with mixed fractions	88	80	85	93	91	80
Pre-reduce fractions when converting to degree measure	63	80	69	86	100	40

Table 3. Students' answers to the question "What criteria can be used to assess the presence of mathematical abilities?"

3 rd year	<ul style="list-style-type: none"> ✓ Logical thinking ✓ Developed imagination ✓ Acuteness ✓ Interest for "Math" discipline 	<ul style="list-style-type: none"> ✓ Fast verbal counting ✓ Ability to simplify ✓ Ability to analyze ✓ Ability to find answers to complex mathematical questions ✓ Strong memorization of material
4 th year	<ul style="list-style-type: none"> ✓ Logical thinking ✓ Critical thinking ✓ Developed memory ✓ Interest for "Math" discipline 	<ul style="list-style-type: none"> ✓ Fast verbal counting ✓ Fast pace of solving tasks ✓ Fast mastering of material ✓ Ability to solve non-standard tasks ✓ Resourcefulness when solving tasks
5 th year	<ul style="list-style-type: none"> ✓ Logical thinking ✓ Critical thinking ✓ Abstract thinking ✓ Spatial thinking ✓ Interest for "Math" discipline 	<ul style="list-style-type: none"> ✓ Fast and correct counting ✓ Ability to rationally solve tasks ✓ Ability to analyze, structure, draw conclusions ✓ Ability to solve tasks of increased complexity ✓ Ability to solve Olympiad tasks ✓ Ability to think outside the box ✓ Ability to see several approaches to solving one task
Master students	<ul style="list-style-type: none"> ✓ Rational thinking ✓ Logical thinking ✓ Critical thinking ✓ Thinking outside the box 	<ul style="list-style-type: none"> ✓ Fast mastering of material ✓ High speed of solving tasks ✓ Fast and high-quality computational skills ✓ Ability to solve tasks of high complexity

✓ Acuteness	✓ Being active in lesson
✓ Ability to generalize and analyze	✓ Willing to know more
✓ Ability to understand abstract mathematical concepts	✓ Ability to solve Olympiad tasks
✓ Involvement into “Math” discipline	

Table 4. Students' answers to the question “How can you develop your math abilities?”

3 rd year	<ul style="list-style-type: none"> ✓ Solving a large number of tasks ✓ Solving smart tasks ✓ Using different methods to solve one task ✓ Self-training in solving mathematical tasks 	<ul style="list-style-type: none"> ✓ Conducting oral counting ✓ Conducting additional classes in mathematics ✓ Using various games (logic, thematic games, chess) ✓ Attending math courses
4 th year	<ul style="list-style-type: none"> ✓ Solving of text tasks ✓ Solving of logical tasks ✓ Solving of Olympiad tasks ✓ Motivation of pupils ✓ Solving of non-standard tasks 	<ul style="list-style-type: none"> ✓ Using math games ✓ Using puzzles ✓ Demonstration of fast counting techniques ✓ Interesting presentation of the topic ✓ Using IT technologies ✓ Attending math courses
5 th year	<ul style="list-style-type: none"> ✓ Statement and solving practically significant tasks for pupil ✓ Solving tasks for development of logical thinking ✓ Solving tasks for development of creative thinking ✓ Solving a task in several ways ✓ Solving Olympiad tasks ✓ Solving tasks of different levels of difficulty 	<ul style="list-style-type: none"> ✓ Using creative and non-standard approaches in teaching ✓ Forming interest for “Math” discipline ✓ Implementing an individual approach ✓ Interesting presentation of material ✓ Extracurricular activities in mathematics ✓ Attending math courses ✓ Participation in mathematical competitions, olympiads ✓ Conducting educational mathematical games in elementary grades ✓ Attending popular science lectures in mathematics
Master students	<ul style="list-style-type: none"> ✓ Solving Olympiad and non-standard tasks ✓ Solving tasks for developing of spatial thinking ✓ Solving tasks of increased complexity ✓ Reading various mathematical literature 	<ul style="list-style-type: none"> ✓ Forming interest for “Math” discipline ✓ Conducting math courses ✓ Conducting extra-curricular thematic mathematical activities ✓ Using educational logic games

Table 5. Students' answers to the question “How are mathematical abilities expressed?”

3 rd year	<ul style="list-style-type: none"> ✓ In speed, accuracy, quality and conciseness of obtaining an answer to task ✓ In ability to think according to the algorithm ✓ In oral task solving (if a pupil solves well orally, then he has the ability) ✓ In presentation of different ways of solving tasks ✓ In ability to solve complex tasks
4 th year	<ul style="list-style-type: none"> ✓ In ability to quickly and correctly reason ✓ In ability to quickly and easily calculate ✓ In ability to think logically ✓ In ability to solve tasks of increased difficulty ✓ In ability to think non-standard ✓ In ability to think abstractly
5 th year	<ul style="list-style-type: none"> ✓ In deliberate and well-grounded decisions ✓ In ability to think logically ✓ In an effort to solve complex tasks ✓ In ability to omit some elementary facts in the chain of reasoning as a matter of course, to do some of the operations in the head without detailed notes ✓ In error-free execution of various types of mathematical operations ✓ In ability to quickly solve examples in the head ✓ In ability to foresee the solution of a mathematical task ✓ In ability to solve tasks in non-standard ways ✓ In spatial thinking ✓ In ability to apply existing knowledge in a new situation (in new conditions)
Master students	<ul style="list-style-type: none"> ✓ In child's passion for mathematics ✓ In ability to think logically and abstractly ✓ In ability to see different ways of solving one task and to choose the most rational of them ✓ In ability to think critically ✓ In developed spatial thinking ✓ In resourcefulness when solving tasks ✓ In ability to abstract

Table 6. Students 'answers to the question "How did your teacher develop your math abilities?"

3 rd year	<ul style="list-style-type: none"> ✓ Solving Olympiad tasks ✓ Conducting extracurricular activities ✓ In the classroom, I gave interesting historical information about the concepts studied
4 th year	<ul style="list-style-type: none"> ✓ Various mathematical competitions ✓ Verbal counting ✓ Self-composing a task according to the proposed drawing ✓ Various logic games ✓ Solving Olympiad tasks
5 th year	<ul style="list-style-type: none"> ✓ Solving logic tasks ✓ Collected volumetric figures from paper (origami) ✓ Solving tasks in 3-4 different ways ✓ Applied game technologies

	<ul style="list-style-type: none"> ✓ Used the method projects in teaching mathematics ✓ Conducted electives in mathematics ✓ In grades 5-6 conducted a course in visual geometry, which contributed to the development of spatial thinking ✓ Conducted "lessons of one task" ✓ Prepared for the Olympiads
Master students	<ul style="list-style-type: none"> ✓ Solving logic tasks ✓ Solving mathematical crosswords ✓ Solving tasks of increased complexity ✓ Solving tasks by non-standard methods ✓ Preparing for Olympiads (solving Olympiad tasks)

Discussion

Analysis of the mathematical part of questionnaires shows that, although there is no purposeful work on special development of mathematical abilities (there are no courses devoted to the study of mathematical abilities of students and their development), students studying at the Faculty of Mathematics gradually and steadily develop these abilities. For some students curtailing the process of mathematical reasoning, generalizing mathematical actions, striving for rational calculations and solutions, search for several ways to solve one problem and for different cases of one problem is irrelevant since this was not paid attention to at school. In the process of studying special and methodological disciplines, these abilities of students develop. In independent academic performance, they, in turn, will develop the mathematical abilities of pupils.

Indeed, in the process of acquiring mathematical knowledge, skills and abilities, mathematical memory, ability to geometric representations, ability to think logically, etc. develop.

1. Capable pupils can independently carry out generalization based on the analysis of one phenomenon, i.e. a specific task is considered by them as an element of a specific class of similar tasks. Getting started with the equation: $\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{2}$, $\sin 3x \cos 2x - \sin 2x \cos 3x = \frac{\sqrt{2}}{2}$, they immediately understand that they have the simplest trigonometric equations in front of them. The rest of pupils should be led to such a generalization by teacher, with the help of a series of appropriately composed tasks.

2. A general algorithm for solving problems of a certain type should be gradually developed. For example, equality to zero of each of the factors of a product equal to zero (while maintaining their meaning). The equation $(\sin^2 x - 3\sin x - 5) \cdot \sqrt{-\cos x} = 0$ is replaced by an equivalent set $\begin{cases} \sin^2 x - 3\sin x - 5 = 0, \\ \cos x = 0 \end{cases}$

with fulfillment of condition $\cos x \leq 0$.

3. Generalization should be carried out not only quickly, but also widely. Pupils can find essential and common in particular, hidden commonality in seemingly different mathematical objects. For example, in the logarithmic equation $3\lg^2 x + 5\lg x - 7 = 0$ or in exponential equation $3^{2x+1} + 5 \cdot 3^x - 7 = 0$ they see ordinary quadratic equation $3x^2 + 5x - 7 = 0$. If this does not happen, then teacher purposefully leads pupils to such a generalization.

4. Methods of solution (solution of atypical non-standard problems) are generalized. For example, finding the radius of a sphere inscribed in a triangular pyramid is reduced to finding the radius of a circle inscribed in a triangle.

Thus, the generalization of mathematical objects, quantitative and spatial relations, actions is gradually being carried out. It can even be said that the ability to generalize generalizations develops, since mathematical symbols, numerical expressions are already products of generalization themselves.

5. Students gradually acquire the ability to vary modes of action, easily switch from one way to another.

Analysis of methodological part of the questionnaires indicates that students have an intuitively correct idea of structure of mathematical abilities, possible methods and forms of organizing work to identify and develop such abilities in pupils. This confirms the point of view of a specialist in the field of mathematics and pedagogy Gnedenko (1991), who believed that "mathematical abilities is much more common than we usually think." But they manifest themselves when pupils are involved in creative, constructive activity. Here, the personality of a teacher comes to the fore, having sufficient competence to identify and develop the abilities of his students.

Conclusion

In the light of the findings of the study, the following recommendations were drawn for future math teachers on developing pupils' mathematical abilities. These guidelines contain professional and psychological components. Revealing the essence of professional component teacher should:

- to know mathematics well (definitions, theorems, rules, methods of solution and proof, etc.);
- to have a wide erudition in order to use capabilities of Mathematics discipline, its connection with the surrounding reality. Show harmony, beauty and logic of mathematical reasoning. Uncover unexpected possibilities of applying certain mathematical facts;

- stimulate pupils' interest in mathematics, increase their motivation for learning, creating a situation of success in solving mathematical problems and stimulating independent educational and cognitive activity of all pupils;
- since the mathematical style of reasoning is inherent in laconicism, pupils should be taught to perform mathematical actions in the most rational ways, while correctly using mathematical symbols;
- future teachers of mathematics should understand that abilities are not given from birth, but develop gradually. Only inclinations are innate. Therefore, it is necessary to develop these inclinations, at best bringing them to abilities;
- all pupils should be developed the ability to master school mathematics course, identify pupils with inclinations for the development of a high level of mathematical abilities;
- development of mathematical abilities is possible only with an appropriate level of development of mathematical memory, therefore, due attention should be paid to this aspect. Conduct regular work on understanding and memorizing methods for solving problems, reasoning schemes, proofs of theorems and statements. At the same time, it should be borne in mind that capable students can independently isolate the element necessary for memorization, while the rest should pay attention to it, emphasize it, and return to it repeatedly;
- not to transfer knowledge in a finished form, create problem situations, rely on subjective experience of pupils, use the method of "expedient tasks";
- pay more attention to independent and research work in the classroom, implement an individual approach;
- when solving new, non-standard problems, avoid negative assessments that have an adverse effect on the student's personality and thereby inhibit the development of mathematical abilities;
- gradually complicate the proposed tasks, provoke the risks of making erroneous decisions, possibility of multiple choice of the course of reasoning, thereby bringing pupils to a higher level - ability to scientific mathematical creativity.

In order to deepen psychological component of training of future mathematics teachers it is necessary to pay attention to the following aspects:

- development of motivational sphere (take into account not only the attractive aspects of the teaching profession, but also realize the significance of its social mission);
- development of volitional sphere (duty, responsibility, discipline);
- development of emotional sphere (feelings of satisfaction and pleasure from working with children, feelings of joy from achieving psychological well-being, and therefore a successful final result);
- development of personality traits necessary for the teaching profession (confidence in the necessity of the chosen profession, pride in the future generation, etc.);
- development of tolerance for criticism in general and in one's own address. Ability to accept someone else's, different from his own, point of view.

It is obvious that success of process of training future teachers for work on development of mathematical abilities depends on many factors. A considerable place in it must be given both to the development of the mathematical abilities of the students themselves and to the development of their pedagogical competencies.

In this regard, it is considered expedient to increase the volume of practical lessons on solving problems of increased complexity, improve methodology for conducting such lessons, develop special courses "Theory of mathematical abilities", "Development of pupils` mathematical abilities ", etc.

During the classes it is necessary to teach students to select non-standard problems (logical, geometric, combinatorial, problems with parameters). Their attention should be paid to the fact that at first teacher examines standard tasks with pupils, checking the availability of necessary subject knowledge and skills on a specific topic. Further, he considers the tasks, when pupils master new theoretical or practical material, accompanying the training with a system of prompts and instructions. After that, he offers tasks for the application of facts obtained by pupils at the second stage. And in conclusion, he gives a task to students to independently compose and solve similar problems.

It is also necessary to pay attention to emotional component of classes, positive emotions should prevail among students. The optimistic component of their future academic performance should become the main one.

We believe that described system of work with future teachers to develop pupils` mathematical abilities, carried out at KFU Nikolai Lobachevsky Institute of Mathematics and Mechanics should bring its results.

References

- Druzhinin, V.N. (1999). *Psychology of general abilities*. Saint-Petersburg: Piter.
- Granberg, C., & Olsson, J. (2015). ICT-supported problem solving and collaborative creative reasoning: Exploring linear functions using dynamic mathematics. *Journal of Mathematical Behaviour*, 37, 48-62.
- Gnedenko, B.V. (1991). Development of thinking and speech in studying mathematics. *Mathematics at school*, 4, 3-9.
- Gusev, V.A. (2003). *Psychological and pedagogical bases of studying math*. Moscow: Verbum-Moscow: Academy.
- Jonsson, B., Norqvist, M., Liljekvist, Y., & Lithner, J. (2014). Learning mathematics through algorithmic and creative reasoning. *Journal of Mathematical behaviour*, 36, 20-32.
- Kolmogorov, A.N. (2001). *About mathematical abilities. Mathematics: Tutorial on history, methodology, didactics*. Moscow: Izd-vo URAO. 239-243.
- Krutetkiy, V.A. (1968). *Psychology of pupils' mathematical abilities*. Moscow.
- Leites, N.S. (2003). *Age endowments and individual differences: selected psychological works*. Voronezh: MODEK; Moscow: MPSI.
- Matyushkin, A.M. & Yakovleva, E.L. (1991). Teacher for gifted. *Social movements and social activity of youth. Proceedings of the All-Union Scientific Conference*, 192-202.
- Morduhay-Boltovskiy, D.D. (1988). *Philosophy. Psychology. Mathematics*. Moscow: Serebryanye niti.
- Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM – The International Journal on Mathematics Education*, 29(3), 75-80.
- Sriraman, B., Haavold, P., & Lee, K. (2013). Mathematical creativity and giftedness: A commentary on and review of theory. *ZDM – The International Journal on Mathematics Education*, 45(2), 215-225.
- Shakirova, K.B., Fazleeva, E.I., & Timerbaeva, N.V. (2020). Beginning Math Teachers' Support. *ARPHA Proceedings IFTE-2020*, 3, 2263-2274.

Teplov, B.M. (1961). *Problems of individual differences*. Moscow: Nauka.

Yakimanskaya I.S. (2004). *Psychological bases of mathematical education: tutorial for students of pedagogical institutions*. Moscow: Izdatelskiy centr "Academy".