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Dynamic Condensation for Reduction of Large-Scale Model

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Abstract. This article discusses model reduction formulations for the computation of large structural models. The simplest method of model reduction is by using static condensation methods. However, this method might not capture the dynamic properties of the structures. A reduction model based on dynamic analysis is performed to reduce the size of structural computation. Assuming that the damping matrix is in proportion to the mass and stiffness matrix, the free vibration analysis is used as a starting point for the structural model analysis. The transformed matrices are obtained by partitioning the matrices in the equations of motion, considering the retained and condensed degrees of freedom. The retained degrees of freedom can be considered as master degrees of freedom, where the size of the system matrices is expected. By several manipulations, the reduced order model is achieved. The computation starts using Guyan's reduction method, and then the system matrices are updated iteratively. The convergence is defined by comparing the eigenvalues of the successive computations. Numerical examples of four and ten-story shear building models are conducted to show the applicability of the methods. The numerical results show that the reduced-order model obtained using this method can predict the actual model's behavior.

Keywords: Reduced-order model, dynamic condensation, static condensation, Guyan reduction method, large-scale model

INTRODUCTION

Many real problems in static and dynamic analysis involve many degrees of freedom (DOF). Finite element models in static or dynamic computations may involve very large DOF. For structures with large DOF, a computational effort will be very demanding, time-consuming, and require big computer storage. Therefore, model reduction is needed to reduce computational efforts. To achieve this purpose, several researchers have proposed static and dynamic condensation. By condensation methods, a reduced-order model is obtained. Reduced-order models have been used in many engineering applications such as structural health monitoring systems, model updating in finite element methods, modal analysis, test-analysis models correlation, vibration control, structural dynamic optimization, and dynamic response analysis [1-2].

Reduced-order models can be achieved by using static or dynamic condensations. The earlier reduced-order model utilized the static condensation technique of Guyan [3] and Irons [4]. In these methods, the inertia force is neglected in the formulation. Therefore, the accuracy of the methods in dynamic problems might be very low. Some researchers improved the static condensation methods to include the inertia effect. Suarez and Singh [5] and Singh and Suarez [6] proposed an iterative dynamic condensation scheme for eigenproblems. Friswell *et al.* [7] extended the Improved Reduced System (IRS) of Callahan [8] to produce an iterative algorithm.

The condensation technique seeks the reduced-order model from the actual or original structural model. The original system matrix is partitioned into submatrices containing retained and condensed degrees of freedom. The retained degrees of freedom can also be considered as the master degrees of freedom, while the condensed degrees of freedom are known as slave degrees of freedom.

This paper considers the application of the dynamic condensation technique to build a reduced-order model of structural systems. Four-story and ten-story shear buildings are considered to show the accuracy of the reduced-order model to predict the slave degrees of freedom that are not modeled in the reduced-order model. The dynamic condensation technique used in this paper follows the dynamic condensation of Weng *et al.* [9][10], which starts the iteration using Guyan transformation matrix. It shows that the method can predict the structural response accurately subject to ground excitations.

STATIC CONDENSATION TECHNIQUE

Consider equations of motion of multi-degree of freedom systems:

$$\mathbf{M}\ddot{\mathbf{U}}(t)+\mathbf{C}\dot{\mathbf{U}}(t)+\mathbf{K}\mathbf{U}(t)=\mathbf{F}(t) \tag{1}$$

In equation 1; **M, C** and **K** is respectively denotes matrices of mass, damping and stiffness. **F** and **U** are respective values for vectors of the force and displacement. The dot represents the derivative with respect to time. Equation (1) can be partitioned into submatrices as:

$$\begin{bmatrix}\mathbf{M}_{mm} & \mathbf{M}_{ms}\\ \mathbf{M}_{ms}^T & \mathbf{M}_{ss}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{U}}_m\\ \ddot{\mathbf{U}}_s\end{Bmatrix}+\begin{bmatrix}\mathbf{C}_{mm} & \mathbf{C}_{ms}\\ \mathbf{C}_{ms}^T & \mathbf{C}_{ss}\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{U}}_m\\ \dot{\mathbf{U}}_s\end{Bmatrix}+\begin{bmatrix}\mathbf{K}_{mm} & \mathbf{K}_{ms}\\ \mathbf{K}_{ms}^T & \mathbf{K}_{ss}\end{bmatrix}\begin{Bmatrix}\mathbf{U}_m\\ \mathbf{U}_s\end{Bmatrix}=\begin{Bmatrix}\mathbf{F}_m\\ \mathbf{F}_s\end{Bmatrix} \tag{2}$$

In equation (2), subscripts *m* and *s* represent master and slave degrees of freedom. The size of the master degree of freedom = n_m and slave degree of freedom = n_s so that the total degree of freedom $n = n_m + n_s$.

In Guyan (1965) method, mass and damping are neglected so that we have a static equation:

$$\begin{bmatrix}\mathbf{K}_{mm} & \mathbf{K}_{ms}\\ \mathbf{K}_{ms}^T & \mathbf{K}_{ss}\end{bmatrix}\begin{Bmatrix}\mathbf{U}_m\\ \mathbf{U}_s\end{Bmatrix}=\begin{Bmatrix}\mathbf{F}_m\\ \mathbf{F}_s\end{Bmatrix} \tag{3}$$

By considering the force in the slave degree of freedom $\mathbf{F}_S=\mathbf{0}$, the displacements of the slave degree of freedom $\mathbf{U}_S$ can be computed from the second submatrix equation as:

$$\mathbf{U}_s=-\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^{\mathrm{T}}\mathbf{U}_m=\mathbf{t}_G\,\mathbf{U}_m \tag{4}$$

Therefore, displacements of the structure according to the Guyan method can be obtained from:

$$\mathbf{U}=\begin{Bmatrix}\mathbf{U}_m\\ \mathbf{U}_s\end{Bmatrix}=\mathbf{T}_G\mathbf{U}_m=\begin{Bmatrix}\mathbf{I}_m\\ \mathbf{t}_G\end{Bmatrix}\mathbf{U}_m \tag{5}$$

By substituting equation (4) into the first row of equation (3), we can obtain:

$$\left(\mathbf{K}_{mm}-\mathbf{K}_{ms}\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^{\mathrm{T}}\right)\mathbf{U}_m=\mathbf{K}_R\,\mathbf{U}_m=\mathbf{F}_m \tag{6}$$

In equation (6), we use the reduced stiffness matrix:

$$\mathbf{K}_R=\mathbf{T}_G^T\,\mathbf{K}\,\mathbf{T}_G \tag{7}$$

Similarly, if we use the same transformation matrix for mass and damping, the reduced mass and damping matrices can be obtained as:

$$\mathbf{M}_R = \mathbf{T}_G^T \mathbf{M} \mathbf{T}_G \tag{8}$$

$$\mathbf{C}_R = \mathbf{T}_G^T \mathbf{C} \mathbf{T}_G \tag{9}$$

In order to obtain the reduced system matrices in Guyan reduction method, equations 7, 8 and 9 are used.

DYNAMIC CONDENSATION METHOD

Consider a free vibration problem and make use of a similar partition as in equation (2); we have:

$$\begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{ms}^T & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}}_m \\ \ddot{\mathbf{U}}_s \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{ms}^T & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_m \\ \mathbf{U}_s \end{Bmatrix} = \mathbf{0} \tag{10}$$

Following the procedure of Weng *et al.* [9][10], the second row of equation (10) results in:

$$\mathbf{U}_s = -\mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T \ddot{\mathbf{U}}_m + \mathbf{M}_{ss} \ddot{\mathbf{U}}_s + \mathbf{K}_{ms}^T \mathbf{U}_m\right) \tag{11}$$

By taking the transformation of the degree of freedom to the master degree of freedom as:

$$\mathbf{U} = \begin{Bmatrix} \mathbf{U}_m \\ \mathbf{U}_s \end{Bmatrix} = \mathbf{T}\,\mathbf{U}_m = \begin{Bmatrix} \mathbf{I}_m \\ \mathbf{T}_t \end{Bmatrix} \mathbf{U}_m \tag{12a}$$

or

$$\mathbf{U}_s = \mathbf{T}_t\, \mathbf{U}_m \tag{12b}$$

Therefore, equation (11) can be written as:

$$\mathbf{T}_t\, \mathbf{U}_m = -\mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss} \mathbf{T}_t\right)\ddot{\mathbf{U}}_m + \mathbf{t}_G \mathbf{U}_m \tag{13}$$

$$\mathbf{t}_G = -\mathbf{K}_{ss}^{-1} \mathbf{K}_{ms}^T \tag{14}$$

We are going to seek the equivalent free vibration problem of the reduced order system as:

$$\mathbf{M}_R \ddot{\mathbf{U}}_m + \mathbf{K}_R \mathbf{U}_m = \mathbf{0} \tag{15}$$

From equation (15), the acceleration of the master degree of freedom becomes:

$$\ddot{\mathbf{U}}_m = -\mathbf{M}_R^{-1}\, \mathbf{K}_R\, \mathbf{U}_m \tag{16}$$

By substituting equation (16) into equation (13) we can obtain:

$$\mathbf{T}_t\, \mathbf{U}_m = \mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss} \mathbf{T}_t\right)\mathbf{M}_R^{-1} \mathbf{K}_R \mathbf{U}_m + \mathbf{t}_G \mathbf{U}_m \tag{17}$$

or

$$\mathbf{T}_t = \mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss} \mathbf{T}_t\right)\mathbf{M}_R^{-1} \mathbf{K}_R + \mathbf{t}_G \tag{18}$$

$$\mathbf{T}_t = \mathbf{t}_d + \mathbf{t}_G \tag{19}$$

where:

$$\mathbf{t}_d = \mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{T}_t\right)\mathbf{M}_R^{-1}\mathbf{K}_R \tag{20}$$

Consider now the stiffness of the reduced system as:

$$\mathbf{K}_R = \mathbf{T}^T \mathbf{K}\mathbf{T} \tag{21}$$

By substituting the transformation matrix $\mathbf{T} = \begin{Bmatrix} \mathbf{I}_m \\ \mathbf{T}_t \end{Bmatrix}$, we can obtain $\mathbf{K}_R = \begin{bmatrix} \mathbf{I}_m & \mathbf{T}_t^T \end{bmatrix} \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{ms}^T & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{I}_m \\ \mathbf{T}_t \end{Bmatrix}$ and finally:

$$\mathbf{K}_R = \mathbf{K}_{mm} + \mathbf{K}_{ms}\mathbf{T}_t + \mathbf{T}_t^T\left(\mathbf{K}_{ms}^T + \mathbf{K}_{ss}\mathbf{T}_t\right) \tag{22a}$$

Substituting equations (19), (20), and (14) into equation (22a), we obtain:

$$\mathbf{K}_R = \mathbf{K}_G + \mathbf{t}_d^T \mathbf{K}_{ss}\,\mathbf{t}_d \tag{22b}$$

where:

$$\mathbf{K}_G = \mathbf{K}_{mm} + \mathbf{K}_{ms}\,\mathbf{t}_G \tag{23}$$

We can use the same procedure for the reduced mass system. The reduced mass system can be written as:

$$\mathbf{M}_R = \mathbf{T}^T \mathbf{M}\mathbf{T} \tag{24}$$

By using the same transformation matrix **T,** $\mathbf{M}_R = \begin{bmatrix} \mathbf{I}_m & \mathbf{T}_t^T \end{bmatrix} \begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{ms}^T & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{I}_m \\ \mathbf{T}_t \end{Bmatrix}$ we can obtain:

$$\mathbf{M}_R = \mathbf{M}_{mm} + \mathbf{M}_{ms}\mathbf{T}_t + \mathbf{T}_t^T\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{T}_t\right) \tag{25}$$

Considering equation (19) we can obtain:

$$\mathbf{M}_R = \mathbf{M}_G + \mathbf{t}_d^T\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{T}_t\right) + \left(\mathbf{M}_{ms} + t_G^T\mathbf{M}_{ss}\right)\mathbf{t}_d \tag{26}$$

where:

$$\mathbf{M}_G = \mathbf{M}_{mm} + \mathbf{M}_{ms}\mathbf{t}_G + \mathbf{t}_G^T\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{t}_G\right) \tag{27}$$

By substituting equations (22) and (26) into equation (15) and considering equations (23), (27), and (19), finally we can obtain:

$$\mathbf{M}_d\ddot{\mathbf{U}}_m + \mathbf{K}_G\mathbf{U}_m = \mathbf{0} \tag{28}$$

where

$$\mathbf{M}_d = \mathbf{M}_G + \left(\mathbf{M}_{ms} + \mathbf{t}_G^T\mathbf{M}_{ss}\right)\mathbf{t}_d \tag{29}$$

Substituting equation (27) into equation (29), we can obtain:

$$\mathbf{M}_d = \mathbf{M}_{mm} + \mathbf{M}_{ms}\mathbf{T}_t + \mathbf{t}_G^T\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}^T\mathbf{T}_t\right) \tag{30}$$

Equation (28) can be considered as the free vibration problem of the reduced-order system, but with the involvement of the transformation matrix $\mathbf{T}_t$. To solve this eigenvalue problem, we do it iteratively so that the resulting eigenvalue fits the result of the reduced order system. From equation (28) we can compute the acceleration of the master degree of freedom as:

$$\ddot{\mathbf{U}}_m = -\mathbf{M}_d^{-1}\mathbf{K}_G\mathbf{U}_m \tag{31}$$

Substituting equation (31) into equation (13) $\mathbf{T}_t\,\mathbf{U}_m = -\mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{T}_t\right)\ddot{\mathbf{U}}_m + \mathbf{t}_G\mathbf{U}_m$ results in:

$$\mathbf{T}_t\,\mathbf{U}_m = \mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{T}_t\right)\mathbf{M}_d^{-1}\mathbf{K}_G\mathbf{U}_m + \mathbf{t}_G\mathbf{U}_m \tag{32}$$

Finally, we can obtain the transformation matrix as:

$$\mathbf{T}_t = \mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{T}_t\right)\mathbf{M}_d^{-1}\mathbf{K}_G + \mathbf{t}_G \tag{33}$$

This transformation matrix in equation (33) is used in equation (30) to solve the eigenvalue system in equation (28). However, as the transformation matrix is still coupled, we must do iteratively. Following Weng *et al.* (2017), the iterative procedure is done as follows:

(a) The first iteration is the Guyan method:

$$\mathbf{T}_t^{[0]} = \mathbf{t}_G = -\mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^T$$
$$\mathbf{M}_d^{[0]} = \mathbf{M}_G = \mathbf{M}_{mm} + \mathbf{M}_{ms}\mathbf{t}_G + \mathbf{t}_G^T\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{t}_G\right)$$
$$\mathbf{K}_G = \mathbf{K}_{mm} + \mathbf{K}_{ms}\,\mathbf{t}_G$$

(b) The transformation matrix $\mathbf{T}_t$ and the mass matrix $\mathbf{M}_d$ are updated iteratively for $k = 1, 2, 3, \ldots$ as follows:

$$\mathbf{T}_t^{[k]} = \mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{T}_t^{[k-1]}\right)\left(\mathbf{M}_d^{[k-1]}\right)^{-1}\mathbf{K}_G + \mathbf{t}_G$$
$$\mathbf{M}_d^{[k]} = \mathbf{M}_{mm} + \mathbf{M}_{ms}\mathbf{T}_t^{[k]} + \mathbf{t}_G^T\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}^T\mathbf{T}_t^{[k]}\right)$$

(c) The iteration process is stopped until the error of eigenvalues $\lambda^{[k]} = eig\left(\left(\mathbf{M}_d^{[k]}\right)^{-1}\mathbf{K}_G\right)$ from the two successive iterations reach $error = \left[\dfrac{\lambda^{[k]} - \lambda^{[k-1]}}{\lambda^{[k-1]}}\right] < tol$

(d) The matrices of the reduced-order system can be achieved as:

$$\mathbf{M}_R = \mathbf{M}_{mm} + \mathbf{M}_{ms}\mathbf{T}_t + \mathbf{T}_t^T\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{T}_t\right)$$
$$\mathbf{K}_R = \mathbf{K}_{mm} + \mathbf{K}_{ms}\mathbf{T}_t + \mathbf{T}_t^T\left(\mathbf{K}_{ms}^T + \mathbf{K}_{ss}\mathbf{T}_t\right)$$
$$\mathbf{C}_R = a_1\mathbf{M}_R + a_2\mathbf{K}_R$$
$$\mathbf{F}_R = \mathbf{F}_m + \mathbf{T}_t^T\mathbf{F}_s$$

An Octave program was developed to automate and carry out the calculation.

APPLICATIONS TO MODEL REDUCTION PROBLEMS

Numerical Example - 1

A four-story building modeled as a shear building, with = $m_2 = m_3 = 542$ tons, $m_4 = 514$ tons, the story stiffness is uniform with $k = 3.5\times10^5$ kN/m for all stories. The damping matrix of the structure is assumed to be in proportion to the stiffness matrix, where the damping ratio is taken to be 2%.

Suppose we are going to reduce the model into two degrees of freedom. The developed program then computes the transformation matrix and the reduced-order system's mass, damping, and stiffness matrices. The error tolerance in this simulation is 0.01, and the maximum iteration is set at 20. The resulting transformation matrix is:

$$\mathbf{T}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2.8432 & 2.8588 \\ -5.2693 & 4.3297 \end{bmatrix}$$

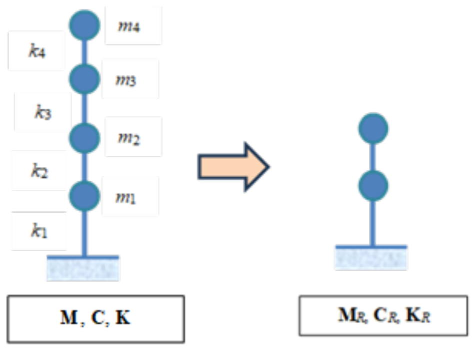


FIGURE 1. Four-story building and reduced-order model

The mass, damping, and stiffness matrices of the reduced-order system are:

$$\mathbf{M}_R=\begin{bmatrix} 19195.08 & -16132.07 \\ -16132.07 & 14607.07 \end{bmatrix}, \mathbf{C}_R=\begin{bmatrix} 25050.80 & -15456.32 \\ -15456.32 & 10382.02 \end{bmatrix}, \mathbf{K}_R=\begin{bmatrix} 5589477.40 & -3448703.06 \\ -3448703.06 & 2316495.04 \end{bmatrix}$$

The reduced-order system is subjected to a ground motion of El Centro 1940 earthquake to validate the results. The response of the slave degrees of freedom is obtained from the master degrees of freedom by utilizing the transformation matrix. The responses are then compared to the ones of the original system. Figure 2 shows the plot of the reduced-order and the original model responses. The displacement response of the slave degrees of freedom of the reduced-order model is computed by using the transformation matrix.

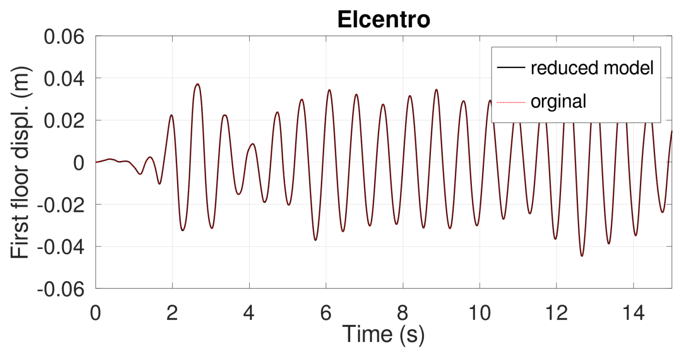
The root-mean-square (RMS) of the response differences of the original and reduced-order models are then computed as shown in Table 1.

TABLE 1. The RMS response differences of example 1

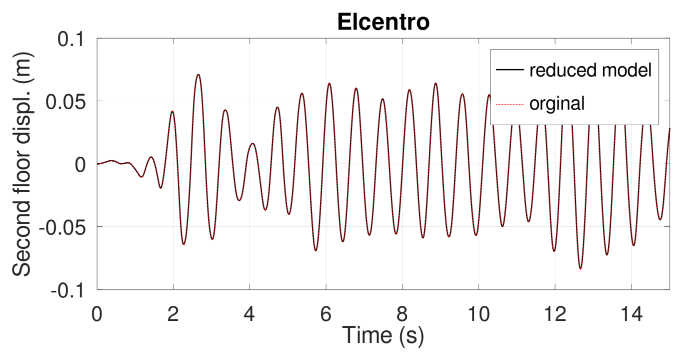
Floor no	RMS response differences
1	1.6839e-04
2	8.6744e-05
3	1.0962e-04
4	9.4746e-05

Numerical Example - 2

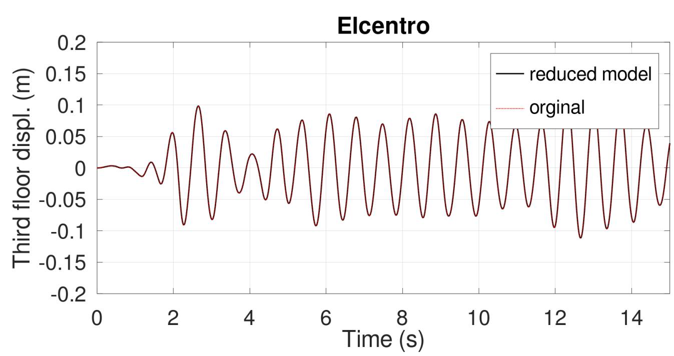
A ten-story shear building discussed in Hadi and Arfiadi [11] is taken for this example. The building properties are shown in Table 2. The damping of the structure is assumed to be in proportion to the stiffness of the structure.



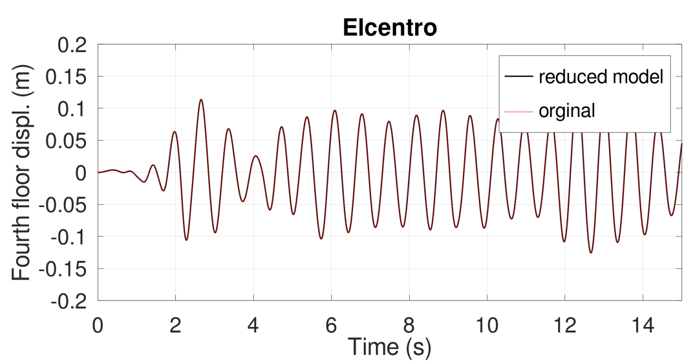
(a) First floor displacement of example 1



(b) Second floor displacement of example 1



(c) Third floor displacement of example 1



(d) Fourth floor displacement of example 1

FIGURE 2. Comparison of the responses of example 1 due to El Centro 1940 earthquake

TABLE 2. Structural properties of building in example 2

Floor/story no	Mass (ton)	Stiffness (kN/m)
1	179	62.47e3
2	170	52.26e3
3	161	56.14e3
4	152	53.02e3
5	143	49.91e3
6	134	46.79e3
7	125	43.67e3
8	116	40.55e3
9	107	37.43e3
10	98	49.91e3;

The structural model is reduced to three degrees of freedom model. A similar program as in Example 1 was used where the tolerance for error in the eigenvalues differences is 0.01 and the maximum iteration is set to 20. The resulting transformation matrix of the slave degrees of freedom to the master degrees of freedom is:

$$\mathbf{T}_t = \begin{bmatrix} 9.9196 & -10.240 & 5.1526 \\ 42.7681 & -39.8629 & 15.2522 \\ 108.8516 & -96.5668 & 33.2850 \\ 208.1254 & -179.6977 & 58.8111 \\ 325.7638 & -276.8911 & 88.0783 \\ 434.0201 & -365.6541 & 114.5114 \\ 500.6159 & -420.0423 & 130.6143 \end{bmatrix}$$

The resulting of the mass, damping, and stiffness matrices of the reduced-order system are:

$$\mathbf{M}_\mathrm{R} = \begin{bmatrix} 64305456.37 & -54394510.09 & 17170826.03 \\ -54394510.09 & 46019781.16 & -14532600.64 \\ 17170826.03 & -14532600.64 & 4593132.40 \end{bmatrix}$$

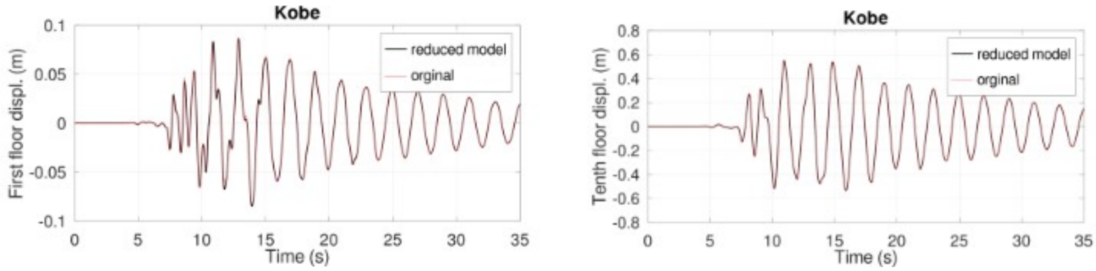
$$\mathbf{C}_R = \begin{bmatrix} 23759385.23 & -19787516.44 & 6032580.27 \\ -19787516.44 & 16490451.72 & -5032418.564 \\ 6032580.27 & -5032418.56 & 1538333.84 \end{bmatrix}$$

$$\mathbf{K}_R = \begin{bmatrix} 1845885691.65 & -1537308020.41 & 468675998.41 \\ -1537308020.41 & 1281156417.89 & -390972633.40 \\ 468675998.41 & -390972633.40 & 119514389.46 \end{bmatrix}$$

The ground motion due to Kobe 1995 earthquake is applied to the original and reduced-order system to evaluate the method's accuracy. The first and the tenth-floor displacements are plotted in Figure 3.

The displacement of the slave degrees of freedom of the reduced-order model is computed by using the transformation matrix. The maximum displacements of each floor from the original model and those computed from the reduced-order model are plotted in Figure 4. Table 3 shows the RMS differences for the displacements of the original and reduced-order systems and the differences for the maximum displacements in each floor.

Similar simulations were carried out to the original and reduced-order model subject to El Centro 1940 ground acceleration. The displacements of the tenth floor of both models are given in Figure 5. The maximum responses are depicted in Figure 6, while the RMS and maximum response differences are shown in Table 4.



(a) First floor displacement of example 2 (b) Tenth floor displacement of example 2

FIGURE 3. Comparison of the responses of example 2 due to Kobe 1995 earthquake

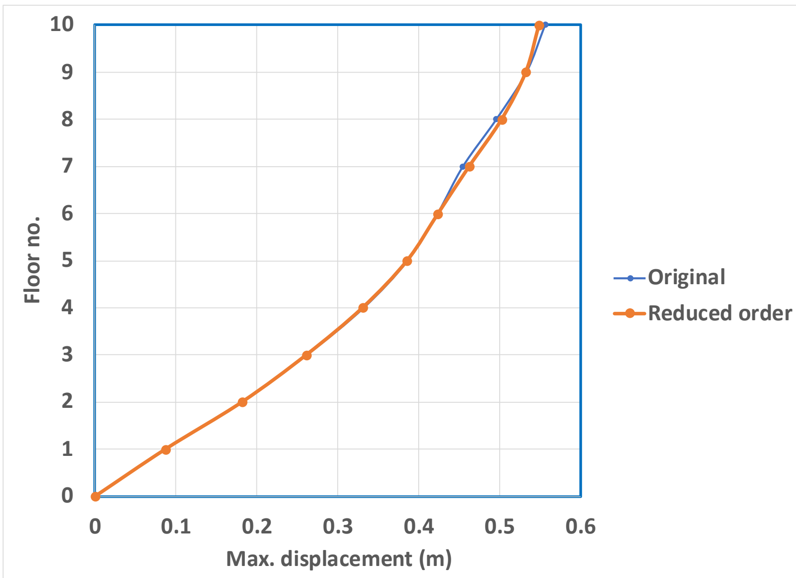


FIGURE 4. Comparison of the maximum response for example 2 due to Kobe 1995 earthquake

TABLE 3. Responses due to Kobe 1995 earthquake of example 2

Floor/story no	RMS differences	Maximum displacement (m)		Maximum responses differences (%)
		Original model	Reduced order model	
1	3.9484e-04	0.087322	0.086896	-0.49
2	1.3167e-03	0.181812	0.181399	-0.23
3	1.1349e-03	0.261913	0.260892	-0.39
4	1.7422e-04	0.332132	0.330319	-0.55
5	1.3462e-03	0.386421	0.385420	-0.26
6	3.9484e-04	0.423318	0.423759	0.10
7	1.3167e-03	0.454683	0.462162	1.64
8	1.1349e-03	0.496008	0.502330	1.27
9	1.7422e-04	0.533224	0.532133	-0.20
10	1.3462e-03	0.556166	0.548452	-1.39

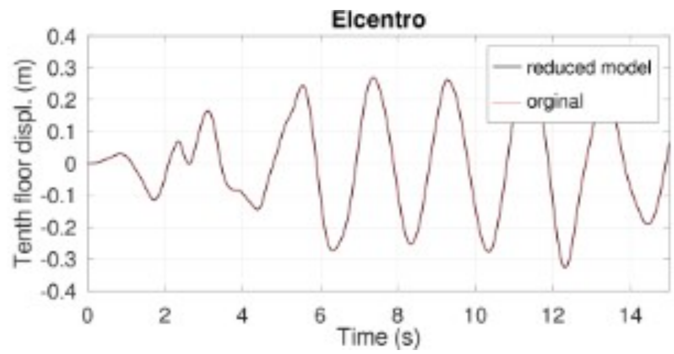


FIGURE 5. Comparison of the responses for example 2 due to El Centro 1940 earthquake

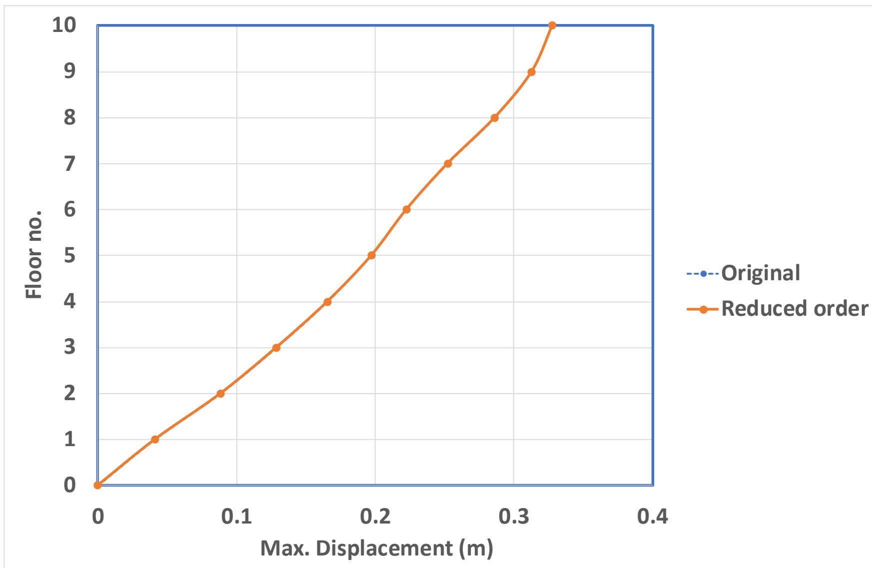


FIGURE 6. Comparison of the maximum response of example 2 due to El Centro 1940 earthquake

TABLE 4. Responses due to El Centro 1940 earthquake of example 2

Floor/story no	RMS differences	Maximum displacement (m)		Maximum responses differences (%)
		Original model	Reduced order model	
1	9.95e-04	0.041069	0.041158	0.22
2	5.65e-04	0.088413	0.088452	0.04
3	3.56e-04	0.128797	0.128696	-0.08
4	6.97e-04	0.165636	0.165558	-0.05
5	4.84e-04	0.197077	0.19717	0.05
6	2.41e-04	0.222343	0.222467	0.06
7	6.06e-04	0.252288	0.252132	-0.06
8	5.09e-04	0.286312	0.286079	-0.08
9	1.01e-04	0.312828	0.312774	-0.02
10	6.05e-04	0.327414	0.327616	0.06

DISCUSSIONS

The procedure for model reduction is verified by the two given numerical examples 1 and 2. The given first example is a four-story building modeled as a shear building so the building has four degrees of freedom. The model for reduced-order is computed by using the Octave program. The displacements of slave degrees of freedom are computed using the transformation matrix obtained from this method and compared to the ones of the original model.

As it can be seen in Figure 2, the reduced-order model can approximate the original model. The time history responses due to El Centro 1940 ground acceleration from the reduced-order model match the original model. A similar conclusion is obtained from the computation of RMS differences of both models, as can be seen in Table 2, where the differences in the responses are very small.

A ten-story building was taken as the second example for evaluation of the method further. The reduced-order model is converted to a three-degrees-of-freedom system. The transformation matrix and the reduced-order model's mass, damping, and stiffness matrix are computed using the same method. To validate this method, simulations of the responses to Kobe 1995 and El Centro 1940 ground excitations are conducted for both the original and reduced-order models. The time history responses from both models are plotted in Figure 3, subjected to Kobe 1995 earthquake, and in Figure 5, subjected to El Centro 1940 earthquake. Due to space limitations, only the first and tenth-floor displacements were shown in Figure 3, and only the tenth-floor displacement was plotted in Figure 5. From those figures, it can be seen that both models fit each other during the time history response. In addition, the maximum responses of both models are plotted in Figures 4 and 6, for Kobe 1995 and El Centro 1940 earthquakes, respectively. Figures 4 and 6 show that the maximum responses of both models fit well. For both ground excitations, the RMS differences and the differences of the maximum responses were computed and shown in Tables 3 and 4, respectively. Tables 3 and 4 show that, due to Kobe 1995 and El Centro 1940 earthquakes, the differences of the RMS time history responses are very small. Similar observations were found for maximum response differences subject to both earthquakes.

CONCLUSION

This paper considers the method of reduction of large structural models by using the technique of dynamic condensation. An iterative solution is presented considering the free vibration problems. An Octave program is then developed to solve the problem. Numerical examples of four- and ten-story buildings are conducted to show the effectiveness of the method. For both examples, the slave degrees of freedom were obtained using the computed transformation matrix. The system matrices, such as mass, damping and stiffness, were obtained accordingly. The responses of both models were then compared to evaluate the accuracy of the procedure. From both examples, it is obtained that the reduced-order model can well fit the original model.

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